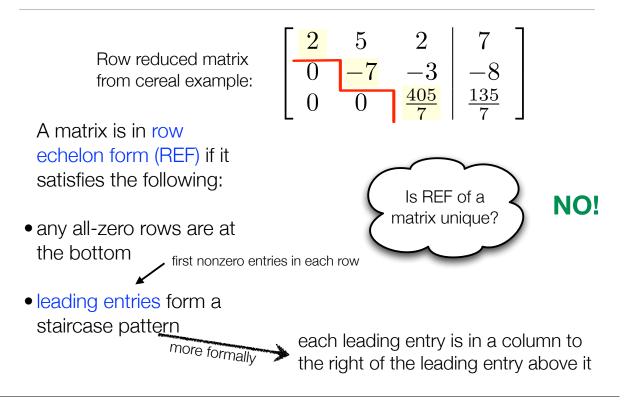
Solving linear systems

Math 40, Introduction to Linear Algebra January 2012

REF -- row echelon form

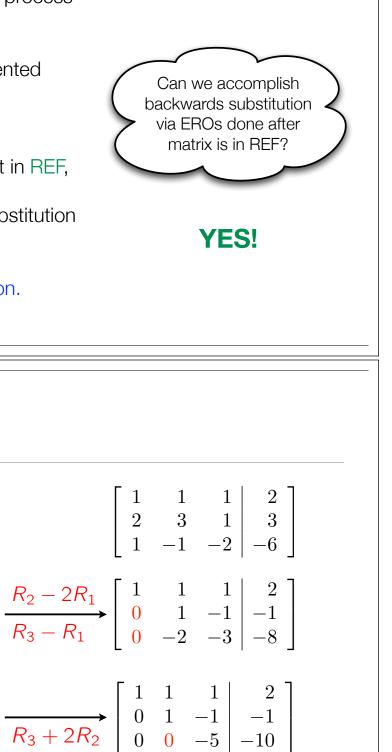


Gaussian elimination

Given a linear system, the process of

- expressing it as an augmented matrix,
- performing EROs on the augmented matrix to get it in REF,
- and, finally, using back substitution to solve the system

is called Gaussian elimination.



matrix is in REF

We could now convert to back to equations and do back substitution...

Example 1

 $x_1 + x_2 + x_3 = 2$

 $2x_1 + 3x_2 + x_3 = 3$

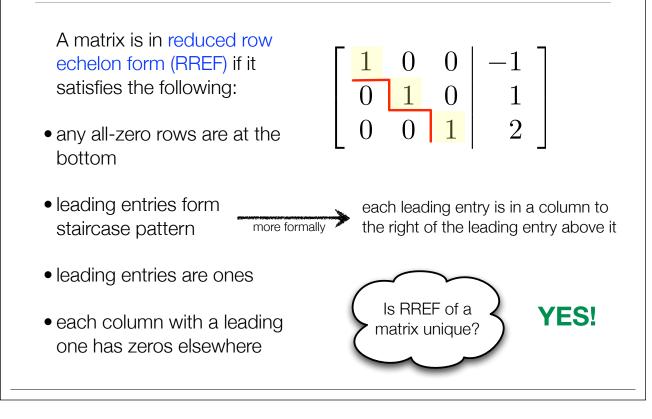
 $x_1 - x_2 - 2x_3 = -6$

rewrite linear system as

augmented matrix

Example 1

 $\begin{array}{c} x_{1} + x_{2} + x_{3} = 2\\ 2x_{1} + 3x_{2} + x_{3} = 3\\ x_{1} - x_{2} - 2x_{3} = -6\\ \text{rewrite linear system as}\\ \text{augmented matrix}\\ \left[\begin{array}{c}1 & 1 & 1 & 1 & 2\\ 0 & 0 & -5 & | & -10\end{array}\right]\\ \hline \begin{array}{c} \hline \\ 1 & 1 & 1 & 1 & 2\\ 0 & 0 & -5 & | & -10\end{array}\right]\\ \hline \begin{array}{c} \hline \\ 1 & 1 & 1 & 1 & 2\\ 0 & 0 & -5 & | & -10\end{array}\right]\\ \hline \begin{array}{c} \hline \\ R_{2} - 2R_{1} \\ R_{3} - R_{1} \end{array} \begin{bmatrix} 1 & 1 & 1 & | & 2\\ 0 & 1 & -1 & | & -1\\ 0 & -2 & -3 & | & -8\end{array}\right]\\ \hline \hline \\ \hline \\ R_{3} + 2R_{2} \end{array} \begin{bmatrix} 1 & 1 & 1 & | & 2\\ 0 & 1 & -1 & | & -1\\ 0 & 0 & -5 & | & -10\end{array}\right]\\ \hline \\ \text{matrix is in REF}\end{array}$



RREF -- reduced row echelon form

A matrix is in reduced row echelon form (RREF) if it satisfies the following:

- any all-zero rows are at the bottom
- leading entries form staircase pattern
- leading entries are ones
- each column with a leading one has zeros elsewhere

Example:

[1	*	0	0	*	0	*	*
0	0	1	0	*	0	*	*
0	0	0	1	*	0	*	*
0	0	0	0	0	1	*	*
0	0	0	0	0	0	0	0
0	* 0 0 0 0 0	0	0	0	0	0	0

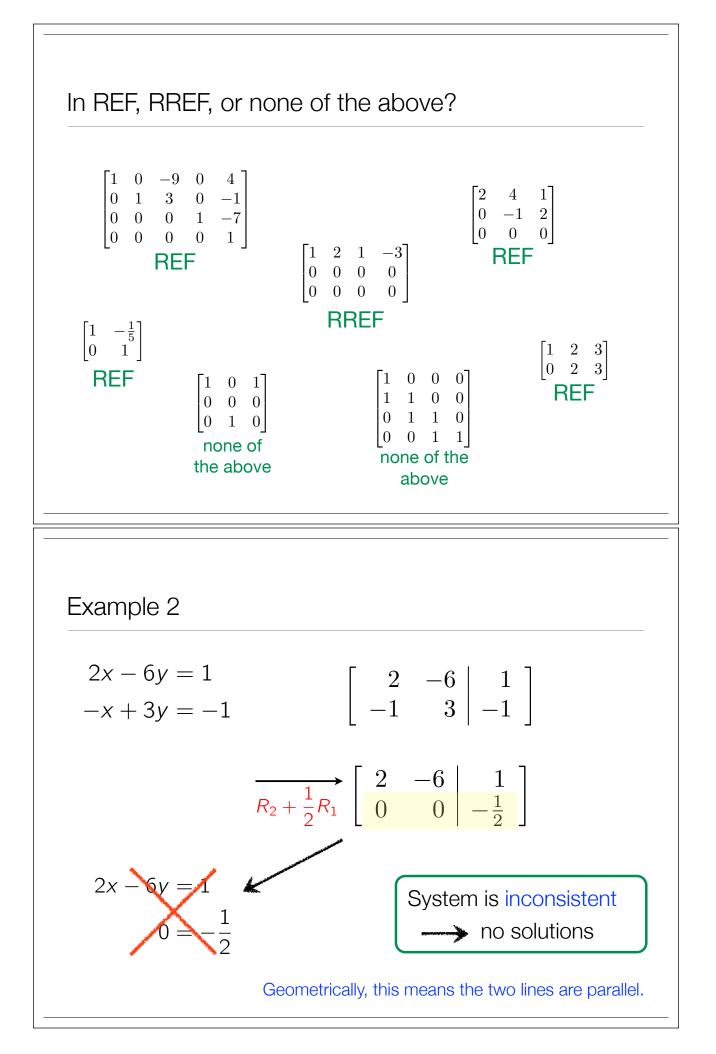
Where must there be zeros, given leading entries above?

Gauss-Jordan elimination

Given a linear system, the process of

- expressing it as an augmented matrix,
- performing EROs on the augmented matrix to get it in RREF,
- and writing the solution to the system

is called Gauss-Jordan elimination.



Example 3

