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University of British Columbia, Vancouver, BC V6T 1Z2, Canada
boyd@math.ubc.ca

Discrete Mathematics: Elementary and Beyond. By László Lovász, József Pelikán, and Katalin Vesztergombi. Springer, New York, 2003, ix + 290 pp. Cloth: ISBN 0-387-95584-4, $69.95. Paper: ISBN 0-387-95585-2, $39.95.

Reviewed by **Arthur T. Benjamin**

A former student of mine once told me that his first discrete mathematics course was like "kindergarten on steroids." It begins with counting (combinatorics) followed by more playing with numbers (number theory), and at the end connects the dots and does some coloring (graph theory). To carry this analogy further, like children entering elementary school, students taking a discrete mathematics course enter with mathematical experience but often without much mathematical maturity. It is the teacher's job to see that the students emerge from the class with a love of learning mathematics.

Typically, students taking their first course in discrete mathematics have calculus in their background, and sometimes linear algebra and differential equations. This course is a great opportunity for students to see that mathematics is more than symbol manipulation, which is how many of them view calculus. Here, students can learn how to read and write rigorous proofs with objects that are easy to grasp and play with. Let's face it: most calculus students struggle to understand the definition of limit and never really understand the proofs that use this sophisticated idea. It is a rare calculus class where students come away with an appreciation of the process and pleasures of rigorous thinking. In linear algebra, the primary tools—vectors—are simpler, but few of the theorems are surprising, and their proofs have a computational flavor to them. By contrast, in discrete mathematics, the primary objects are sets, numbers, and graphs, which the students immediately understand. No matter how hard I try to build up the "Invertible Matrix Theorem" in my linear algebra class, the students' reaction to it pales in comparison with their reaction to the derangements problem, Fermat's little theorem, or the 5-color theorem. Not only are the theorems in discrete mathematics surprising and beautiful, the students are excited about the fact that they understand how to derive them. In a discrete mathematics class, one almost cannot avoid exposing students to open problems along the way.

When teaching a one-semester course in discrete mathematics, I prefer to avoid topics that place too much emphasis on symbol manipulation, such as Boolean algebra, truth tables, and methods for solving linear recurrences. Topics should begin with an intriguing problem and end with a satisfying application. My goal is for each class to have something so intriguing that students are eager to tell their friends what they learned in lecture today.

Discrete Mathematics: Elementary and Beyond covers nearly every topic that I do, and more. Its distinguished group of authors write in a very friendly style that sounds like a professor speaking one-on-one with a student in her office and is laced with humor and interesting side topics. For example, as an illustration of the pigeonhole principle, one problem on page 35 states, "We shoot 50 shots at a square target, the side of which is 70 cm long. We are quite a good shot, because all of our shots hit the target. Prove that there are two bulletholes that are closer than 15 cm." The solution begins, "Imagine that our target is an old chessboard. One row and one column of it has fallen off, so it has 49 squares." (It goes on to show rigorously that the greatest distance between two points inside a square is the length of the diagonal.)

The book begins with a preliminary chapter on sets and counting, with a group of mathematical party-goers providing the motivating problems. In the next four chapters, the authors introduce proof techniques and combinatorial tools: induction, inclusion-exclusion, pigeonhole principle, binomial theorem, combinatorial proofs, and multinomial coefficients. Particular attention is paid to applications of binomial coefficients and their asymptotic analysis. Separate chapters are devoted to combinatorial probability and Fibonacci numbers.

Another nice feature of this book is that most of the identities are proved in more than one way. For instance, a pattern arising in Pascal's Triangle might be proved by induction, by the binomial theorem, and by a combinatorial argument. Consider the Fibonacci identity

$$F_{n-1}^2 + F_n^2 = F_{2n-1}$$

for $n \geq 1$, where $F_0 = 0$, $F_1 = 1$. The authors derive this using *simultaneous induction*, proving that

$$F_n F_{n-1} + F_{n+1} F_n = F_{2n}$$

at the same time. Later, they introduce "modified Fibonacci numbers" defined by $E_0 = a$, $E_1 = b$, and $E_n = E_{n-1} + E_{n-2}$ for $n \geq 2$. The sequence begins $a,\ b,\ a + b,\ a + 2b,\ 2a + 3b,\ 3a + 5b,\ 5a + 8b, \dots$, and it is easy to conjecture and prove (by ordinary induction) that $E_n = F_{n-1}a + F_n b$. Choosing a and b to be Fibonacci numbers leads to the earlier pair of identities and other generalizations. Then, without using induction or generating functions, they give a natural derivation of Binet's formula:

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right].$$

The book devotes two chapters to number theory and applications. It covers just the right amount of number theory to enable the authors to derive the RSA algorithm for public key cryptography with digital signatures. Along the way, it exposes the reader to the prime number theorem, modular arithmentic, Fermat's little theorem, Carmichael numbers, and the computational complexity of the Euclidean algorithm.

I especially liked the coverage of graph theory, with six chapters on topics including trees, greedy algorithms, the traveling salesman problem, planarity, matching theory, and map coloring. It is a rare introductory textbook that derives Cayley's result that the number of labeled trees on n vertices is n^{n-2} and proves that for $n > 30$ the number T_n of unlabeled trees satisfies $2^n \leq T_n \leq 4^n$. The book devotes two chapters to geometry, one of which is titled “Finite Geometries, Codes, Latin Squares, and Other Pretty Creatures.”

The book has many interesting exercises, more than 150 of which are not answered in the back. (I would have liked to have seen even more unanswered exercises to use as potential homework problems.) As its title suggests, the book begins with elementary problems and concepts, and by the end of each chapter, without requiring as much effort as one might have expected, it takes one considerably further. I highly recommend this book for all budding mathematicians and computer scientists.

Harvey Mudd College, Claremont, CA 91711
benjamin@hmc.edu