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Localization of Optimal Strategies in Certain Games

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Assume the payoffs of a matrix game are concave in the index of the maximizing player. That player is shown to have an optimal strategy which uses at most two consecutive pure strategies, identifiable through approximate solution of a related continuous game. Generalizations are given, and the results are applied to a motivating hidden-target model due to Shapley. © 1994 John Wiley & Sons, Inc.

1. INTRODUCTION

Initially, consider a $(1 + m) \times 2$ matrix game $A = [a(r, j)]$ whose rows are indexed by the integers $r \in \{0, 1, \ldots, m\}$, and suppose that for each $j \in \{1, 2\}$ the function $a(r, j)$ is concave in r; i.e.,

$$a(r + 1, j) + a(r - 1, j) \leq 2a(r, j) \qquad (0 < r < m). \tag{1}$$

The kernel theory of extreme optimal strategies for matrix games tells us that Player 1 (the row player) has an optimal strategy which uses at most two of his pure strategies. It is reasonable to hope, however, that the further assumption (1) of concavity might be useful in determining *which* two optimal strategies are involved.

To be specific, suppose that for $j \in \{1, 2\}$ the functions $a(r, j)$ have natural extensions $b(r, j)$ as continuous functions of a *continuous* variable r, concave over the interval $[0, m]$. For $0 \leq x \leq 1$ and $0 \leq u \leq 1$, set

$$M_b(x, u) = ub(mx, 1) + (1 - u)b(mx, 2). \tag{2}$$

Then M_b can be regarded as the payoff function of a continuous game G_b on the (unit) square. Since $M_b(x, u)$ is concave in x for each value of u, game G_b has a *pure* optimal strategy x_b for Player 1. We now perform the rescaling $r_b = mx_b$, and define the integer $\rho_b = \lfloor r_b \rfloor$. It is plausible to expect that if r_b is integer (i.e., $r_b = \rho_b$), then r_b is also an optimal pure strategy for the original matrix game, while if r_b is noninteger, then in the matrix game Player 1 has an optimal strategy which mixes only the pure strategies ρ_b and $\rho_b + 1$.

If this expectation is correct, then the continuous game G_b might be solved, at least to sufficient accuracy to determine ρ_b; this would identify a 2×2 subgame of A, involving rows ρ_p and $\rho_b + 1$, whose solution would yield a solution of A. For smooth functions $b(\cdot, j)$, the solution process for G_b is likely to be quasianalytical (i.e., calculus based); such a technique is thus especially valuable when the original data $a(r, j)$ involve pa-

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rameters whose role in the optimal strategies is to be elucidated, so that a purely numerical approach is undesirable. And even if no effort to solve G_b is made, the information that Player 1 has in A an optimal strategy which mixes at most two *consecutive* pure strategies would drastically reduce the number of 2×2 subgames of A to be considered.

The procedure described in the last paragraph's first sentence was in fact followed (without proof) in an analysis, given by Shapley [10] and reported by Dresher [1], of an interesting hidden target model. The present work was motivated by desire to pin down explicitly the logic of this plausible analysis. In the next section we develop that logic precisely, for situations considerably more general than 2-column matrix games. In the final section, we apply the results to a reconsideration of the hidden-target model. A more demanding utilization, of both-player versions of our results to solve an N-card variant of the classical [6, p. 101] card game Le Her, will be reported separately to preserve brevity here. We hope and expect that these results will prove applicable to a number of other matrix-game models.

2. ANALYSIS

Instead of passing at once to the most general situation to be considered, we begin with a $(1 + m) \times n$ matrix game $A = [a(r, j)]$ with rows indexed by $r \in I_m = \{0, 1, \ldots, m\}$. Assume that for each column index $j \in \{1, 2, \ldots, n\}$ the function $a(r, j)$ obeys the condition (1) of concavity in r. Further, assume as before that for each j, $b(r, j)$ is a continuous concave function defined on the continuous interval $[0, m]$, which interpolates $a(r, j)$ at the integer points of that interval.

Let Y be the simplex of n-component mixed strategies $y = (y_1, \ldots, y_n)$ for Player 2 in game A. Then in analogy with (2), we can define a function M_b: $[0, 1] \times Y \rightarrow \mathbf{R}$ by

$$M_b(x, y) = \sum_1^n b(mx, j) y_j, \tag{3}$$

and regard M_b as the payoff function of a two-person zero-sum game $G_b = ([0, 1], Y, M_b)$. Since $M_b(x, y)$ is concave in x for fixed y and is convex (indeed, linear) in y for fixed x, we can invoke an appropriately general minimax theorem (e.g., Theorem 1(ii) of Fan [2]) to assert that G_b possesses pure optimal strategies (x_b, y_b) for both players. As before, let $r_b = mx_b$ and $\rho_p = \lfloor r_b \rfloor$. Then we have the following easy first result.

THEOREM 1: If r_b is integral then in the matrix game A, r_b is a pure optimal strategy for Player 1 and y_b is an optimal strategy for Player 2.

PROOF: With the notation

$$v = \sum_1^n a(r_b, j) y_{bj},$$

it suffices to show that

$$\sum_1^n a(r, k) y_{bk} \leq v \leq a(r_b, j),$$

for all (r, j). With e_j denoting the jth n-dimensional unit vector, these conditions are equivalent via (3) to

$$M_b(r/m, y_b) \leq M_b(x_b, y_b) \leq M_b(x_b, e_j),$$

for all (r, j) (here r is discrete), which is true since (x_b, y_b) is a saddle point of game G_b. ■

To extend the analysis to cover the possibility of nonintegral r_b we introduce, in addition to the given general concave interpolators $b(r, j)$ of the data $a(r, j)$, the piecewise-linear interpolators of these data. Specifically, for each j let $\lambda(r, j)$ be the function defined on $[0, m]$ which interpolates *linearly* between the successive data points $(i, a(i, j))$ and $(i + 1, a(i + 1, j))$ for $i \in \{0, 1, \ldots, m - 1\}$. These functions are continuous and concave. They define a continuous game G_λ, which like G_b has at least one saddle point: (x_λ, y_λ). Analogous to r_b and ρ_b, we define $r_\lambda = mx_\lambda$ and ρ_λ and $\rho_\lambda = \lfloor r_\lambda \rfloor$.

The essential device in the following analysis is a reduction from the general case G_b to the particular case G_λ. For clarity, we therefore emphasize that this reduction is proof theoretic rather than algorithmic; we would not expect the game G_λ to make an explicit appearance when applying this article's ideas to the solution of a matrix game A.

THEOREM 2B: In the matrix game A, Player 1 has an optimal strategy which mixes only pure strategies $\lfloor r_b \rfloor$ and $\lfloor r_b \rfloor$.

THEOREM 2L: If r_λ is noninteger, then in the matrix game A Player 1 has an optimal strategy mixing pure strategies $\lfloor r_\lambda \rfloor = \rho_\lambda$ and $\lceil r_\lambda \rceil = \rho_\lambda + 1$ with respective weights $\rho_\lambda + 1 - r_\lambda$ and $r_\lambda - \rho_\lambda$. Player 2 has y_λ as optimal strategy in A, and G_λ and A have the same value.

PROOF (of Theorem 2B): We show that Theorem 2B follows from a special case: The first assertion of Theorem 2L. For this purpose, first note that the postulated properties of b and λ imply that for each j, $b(r, j) \geq \lambda(r, j)$ holds for each subinterval $[i, i + 1]$ of $[0, m]$ and hence on the entire interval $[0, m]$, with equality at the points of I_m. It follows that the functions B, L: $[0, m] \rightarrow \mathbf{R}$ defined by

$$B(r) = \min_j b(r, j), \qquad L(r) = \min_j \lambda(r, j)$$

satisfy $B \geq L$, with equality at the points of I_m. The sets of pure optimal strategies for Player 1, in the respective games G_b and G_λ, can be characterized as arg max(B) and arg max(L). Since B and L are continuous concave functions, their sets of maximizers are nonempty (though possibly degenerate) closed subintervals of $[0, m]$. In particular, we write

$$\arg\max(L) = [c_\lambda, d_\lambda],$$

and define the open interval

$$J_\lambda = (\lceil c_\lambda \rceil - 1, \lfloor d_\lambda \rfloor + 1);$$

i.e., the left endpoint of J_λ is $c_\lambda - 1$ or $\lfloor c_\lambda \rfloor$ according as c_λ is integer or not, and similarly for the right endpoint ($d_\lambda + 1$ or $\lceil d_\lambda \rceil$).

The desired relation between the pure optimal strategies of G_b and those of G_λ is given by

$$\arg\max(B) \subseteq J_\lambda \cap [0, m]. \tag{4}$$

To prove it, we will show that

$$\arg\max(B) \subseteq (\lceil c_\lambda \rceil - 1, m]; \tag{5}$$

by symmetry it will follow that $\arg\max(B) \subseteq [0, \lfloor d_\lambda \rfloor + 1)$, which together with (5) implies (4).

The proof of (5) involves three cases. If $c_\lambda \notin I_m$, then $\lceil c_\lambda \rceil \notin \arg\max(L)$, so that

$$B(c_\lambda) \geq L(c_\lambda) > L(\lceil c_\lambda \rceil) = B(\lceil c_\lambda \rceil);$$

from $B(c_\lambda) > B(\lceil c_\lambda \rceil)$ and the concavity of B it follows that $B(r) < B(c_\lambda)$ for all $r \in [0, \lceil c_\lambda \rceil]$, verifying (5) in this case. If $c_\lambda \in I_m - \{0\}$ then because $c_\lambda - 1 \notin \arg\max(L)$, we have

$$B(c_\lambda) = L(c_\lambda) > L(c_\lambda - 1) = B(c_\lambda - 1),$$

which by the concavity of B implies $B(r) < B(c_\lambda)$ for all $r \in [0, c_\lambda - 1]$, again verifying (5). And if $c_\lambda = 0$, (5) is trivially true.

With (4) now verified, we turn to the conclusion of the Theorem (2B). If r_b is integral, this follows from Theorem 1, so assume r_b noninteger with $\rho_b = \lfloor r_b \rfloor$. By (4), $r_b \in (\lceil c_\lambda \rceil - 1, \lfloor d_\lambda \rfloor + 1)$. Since $r_b > \lceil c_\lambda \rceil - 1$ implies $c_\lambda \leq \lceil r_b \rceil$, and $r_b < \lfloor d_\lambda \rfloor + 1$ implies $\lfloor r_b \rfloor \leq d_\lambda$, it follows that the intervals $[\lfloor r_b \rfloor, \lceil r_b \rceil]$ and $[c_\lambda, d_\lambda]$ must intersect, i.e., that interval $[\rho_b, \rho_b + 1]$ contains at least one $r_\lambda \in \arg\max(L)$. The result now follows from Theorem 2L (or, if r_λ is integer, from Theorem 1 applied to G_λ). ■

The last theorem justifies the procedure described in the introduction, extended from two-row matrix games to *general* matrix games with the postulated concavity property. From either it or Theorem 2L (whose proof is left for last), we have the following.

COROLLARY: In matrix game A, under the concavity assumption (1), Player 1 has an optimal strategy which is a mixture of at most two consecutive pure strategies.

A consequence of the corollary is that, even without solving any continuous game, an optimal strategy for Player 1 in A can be determined after solving m $2 \times n$ games, each involving a pair of *consecutive* rows of the original matrix. Solving a $2 \times n$ game involves maximizing the minimum of n linear functions over an interval, and it is appropriate to note that such a minimum can be determined in $O(n \log n)$ time, as is shown in the Appendix of Megiddo [7].

The previous results can be generalized beyond the class of matrix games. As before, let $I_m = \{0, 1, \ldots, m\}$, and now let Y be any compact convex finite-dimensional polyhedron. Consider a game $G_a = (I_m, Y, a)$ where a: $I_m \times Y \rightarrow \mathbf{R}$ has $a(r, y)$ concave in r for each $y \in Y$, and continuous and convex in y for each $r \in I_m$. Also consider any continuous function b: $[0, m] \times Y \rightarrow \mathbf{R}$ such that for each $r \in I_m$, $b(r, y)$ is convex in

y, while for each $y \in Y$, $b(r, y)$ is concave in $r \in [0, m]$ and interpolates $a(r, y)$ for all $r \in I_m$. [Note that one choice for b is the piecewise-*linear* interpolator $\lambda(r, y)$.] Then with (3) generalized to $M_b(x, y) = b(mx, y)$, and y replacing j and e_j at appropriate points, the preceding arguments still go through, and so the conclusions of Theorems 1 and 2 and the Corollary still remain valid for the Player 1 mixed extension of the game G_a. Further lines of generalization are sketched in the appendix.

Finally, we provide a proof.

PROOF (of Theorem 2L): With the notation

$$v = (\rho_\lambda + 1 - r_\lambda) \sum_1^n a(\rho_\lambda, k) y_{\lambda k} + (r_\lambda - \rho_\lambda) \sum_1^n a(\rho_\lambda + 1, k) y_{\lambda k},$$

it suffices to show that

$$\sum_1^n a(r, k) y_{\lambda k} \le v \le (\rho_\lambda + 1 - r_\lambda) a(\rho_\lambda, j) + (r_\lambda - \rho_\lambda) a(\rho_\lambda + 1, j), \tag{6}$$

for all (r, j). Using the identity

$$r_b = (\rho_b + 1 - r_b)\rho_b + (r_b - \rho_b)(\rho_b + 1),$$

specialized to $b = \lambda$, and the piecewise linearity in r of each $\lambda(r, j)$, we have

$$\lambda(r_\lambda, j) = (\rho_\lambda + 1 - r_\lambda) a(\rho_\lambda, j) + (r_\lambda - \rho_\lambda) a)(\rho_\lambda + 1, j),$$

and so (3) yields, for all $y \in Y$,

$$M_\lambda(x_\lambda, y) = (\rho_\lambda + 1 - r_\lambda) M_\lambda(\rho_\lambda / m, y) + (r_\lambda - \rho_\lambda) M_\lambda((\rho_\lambda + 1)/m, y).$$

This implies that $v = M_\lambda(x_\lambda, y_\lambda)$. It also shows that (6) is equivalent to

$$M_\lambda(r/m, y_\lambda) \le M_\lambda(x_\lambda, y_\lambda) \le M_\lambda(x_\lambda, e_j)$$

for all (r, j), which is true since (x_λ, y_λ) is a saddle-point of game G_λ. ■

An alternative treatment of Theorem 2L has been given by J.A. Filar (informal communication, 1984) and Howard [5].

3. EXAMPLE: THE HIDDEN TARGET

We now revert to the motivating hidden-target model mentioned in the introduction. It involves two aircraft, A_1 and A_2, flying in a formation such that attacking A_1 requires the attacker to cross the field of fire of the protector A_2. Player 2 has chosen which one of these two aircraft is to carry the bomb, i.e., the hidden target; he therefore has two pure strategies.

Player 1, not knowing whether A_1 or A_2 is the bomb carrier, directs a sequence of attacks on the formation by m fighters, one attack per fighter. Each fighter attacks either

A_1 or A_2; if it attacks A_2, its probability of success is $\beta \in (0, 1)$, but if it attacks A_1 while the protector A_2 still survives, then this probability is only $\gamma \in (0, \beta)$. (Once A_2 is destroyed, an attack on A_1 has probability β of success.) Both players know the values of m, β, and γ.

A pure strategy for Player 1 is thus an m-letter word in the alphabet {1, 2}; the ith letter of the word is k if the strategy directs the ith fighter to attack A_k should both targets still survive (otherwise it attacks the single survivor, if any). The payoff to Player 1 in this zero-sum game is his probability of destroying the bomb carrier.

Player 1 has 2^m pure strategies. It is not difficult to check, however, that as asserted in Dresher [1, p. 70], any such strategy calling for precisely r attacks on A_2 is dominated by the strategy which directs the first r attacks to A_2 and the remaining $m - r$ to A_1. Thus the payoff matrix can be taken to have $1 + m$ rows, the rth row for $r \in I_m$ corresponding to choosing r as the number of preplanned attacks on A_2 which precede a switch to A_1 as the preplanned target for the balance of the engagement.

Simple combinatorial-probability arguments and geometric-progression summations show that the entries of row r of the payoff matrix are given, in terms of $R = (1 - \gamma)/(1 - \beta) > 1$, by

$$a(r, 1) = 1 - r\beta(1 - \beta)^{m-1} - (1 - \gamma)^m R^{-r}, \tag{7}$$

$$a(r, 2) = 1 + \gamma(\beta - \gamma)^{-1}(1 - \beta)^m - \beta(\beta - \gamma)^{-1}(1 - \gamma)^m R^{-r}. \tag{8}$$

The formulas in (7) and (8) also make sense for continuous $r \in [0, m]$, providing natural continuous extensions $b(r, j)$ of $a(r, j)$ to this interval for $j \in \{1, 2\}$. Calculations of second derivatives show these functions to be strictly concave. The cited analyses of this model now argue as in the introduction, concluding that the exact solution is to be found by solving the 2×2 subgame on rows ρ_b and $\rho_b + 1$, where $\rho_b = \lfloor r_b \rfloor$, $r_b = mx_b$, and x_b is the pure optimal strategy (unique, thanks to strict concavity) for Player 1 in the derived game G_b on the square. Theorem 2B in the last section provides full justification for that argument.

To reinforce and illustrate this article's general point that the solution process for G_b would not typically be purely numerical, we exhume/adapt from [10] some specifics, for this particular model, that are omitted in the more accessible [1]. It will be convenient to work with the rectangle $[0, m] \times [0, 1]$ instead of the unit square, so that (2) is replaced by

$$M_b(r, u) = ub(r, 1) + (1 - u)b(r, 2). \tag{9}$$

We first seek a saddle point (r_b, u_b) of G_b for which $u_b \in \{0, 1\}$. The endpoints $\{0, m\}$ of $[0, m]$ can be eliminated, as possible choices for r_b, as follows. The pair $(r, u) = (m, 0)$ is ruled out as a saddle point because $b(m, 2) > b(m, 1)$, the pair $(m, 1)$ because $b(r, 1)$ is decreasing at $r = m$. The pair $(r, u) = (0, 1)$ is ruled out as a saddle point because $b(0, 1) > b(0, 2)$, the pair $(0, 0)$ because $b(r, 2)$ is increasing in r.

This last reason also rules out the existence of saddle points (r, u) with $0 < r < m$ and $u = 0$. So on the horizontal boundaries of the rectangle, the only remaining possibilities are points $(r, 1)$ with $0 < r < m$. Such a saddle point would have to satisfy $\partial b(r, 1)/\partial r = 0$, or equivalently r must have the value

$$r^* = m - [\log\{\beta/(1 - \beta)\} - \log\log R]/\log R. \tag{10}$$

Conversely (a nicety of logic omitted from [10] and [1]), if $0 < r^* < m$ and also $b(r^*, 1) \leq b(r^*, 2)$, then by the concavity in r and linearity in u of (9), $(r^*, 1)$ is indeed a saddle point of G_b, and we can set $r_b = r^*$. Otherwise, the unique r_b must belong to a saddle point (r_b, u_b) with $0 < u_b < 1$, and must therefore satisfy the condition $b(r, 1) = b(r, 2)$. This condition yields the transcendental equation

$$r\beta(\beta - \gamma)/\gamma(1 - \beta) = R^{m-r} - 1, \tag{11}$$

where the final (-1) matches Eq. (11) of [10], but is missing from the corresponding equation in [1]. The difference of the two sides of (11) is continuous and monotone over $[0, m]$, with different signs at the endpoints. Thus (11) has a unique root r^*, which under the stated circumstances must give the desired r_b. When the protector A_2 is relatively ineffective so that $R - 1 = \beta(\beta - \gamma)/(1 - \beta) << 1$, then as noted in [10], keeping only the first two terms of the binomial expansion $(1 + (R - 1))^{m-r}$ on the right in (11), suggests the approximate solution

$$r = m\gamma/(\beta + \gamma).$$

APPENDIX: POSSIBLE FURTHER GENERALIZATIONS

Further generalizations of Theorem 2's Corollary might be sought in three directions. One is to weaken the concavity hypothesis (1), presumably weakening the at most two in the corollary's conclusion. For a continuous analog, see Glicksberg [3]. A second is to permit Player 1 a pure-strategy space more general than I_m; then the consecutive in the corollary's statement would be replaced by some more general relation. The third is to permit Player 2 a pure-strategy space Y still more general than in the second paragraph after the Corollary.

Here we will pursue only the third of these directions, and even that rather briefly. Namely, suppose first that Y is a compact convex subset of some Hausdorff linear topological space. Then the previously cited minimax theorem of Fan [2] is general enough that the assertions after the corollary still follow. For possibilities of weakening the *compactness* assumption on Y, see Ha [4]. Alternatively, to drop the convexity assumption, let us now assume of the original game $G_a = (I_m, Y, a)$ only that Y is a compact metric space, that $a(r, y)$ is concave in r for each $y \in Y$, and that $a(r, y)$ is continuous in y for each $r \in I_m$. Assume that function b: $[0, m] \times Y \rightarrow \mathbf{R}$ is for each $r \in [0, m]$ continuous in y, and for each $y \in Y$ is continuous and concave in r and interpolates $a(r, y)$ at all $r \in I_m$. As before, define $M_b(x, y) = b(mx, y)$ and the game $G_b = ([0, 1], Y, b)$. Consider any σ-field $\mathcal{F}$ of subsets of Y, including the individual points of Y (pure strategies for Player 2), such that

$$b^*(r, \mu) = \int_Y b(r, y)\, d\mu(y)$$

exists for every $r \in [0, m]$ and every member μ of the set Y^* of probability distributions over $(Y, \mathcal{F})$; thus $G_b^* = ([0, m], Y^*, b^*)$ is a Player 2 mixed extension of G_b. Suppose further that Y^* admits a topology under which it is compact Hausdorff, and such that b^* is continuous in the second variable (it is automatically continuous in the first variable). Then the previously cited minimax theorem will apply to G_b^*, and with sums replaced by integrals over Y and y_b replaced by an optimal $\mu_b \in Y^*$, the previous arguments carry over to yield the conclusions of Theorems 1 and 2 and of the corollary for the *two*-player mixed extension of G_a. Since that extension depends on $\mathscr{F}$ (i.e., on what constitutes a "mixed strategy" for Player 2), it would be desirable if $\mathscr{F}$ did not depend on *which* concave extension b of the original payoff function a was chosen. All this can be accomplished in a natural way by choosing $\mathscr{F}$ to consist of the Borel subsets of Y, and by employing the weak topology on the resulting set Y^* of mixed strategies; see Section II.6, especially Theorem 6.4, of Parthasarathy [8]. In closing this technical digression, we remark only (a) that the cited minimax theorem may permit relaxation of the continuity assumptions to upper semicontinuity in r and lower semicontinuity in y, and (b) that still further generalization might be obtainable using even more general minimax theorems such as those in Chapter 5 of Parthasarathy and Raghavan [9].

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