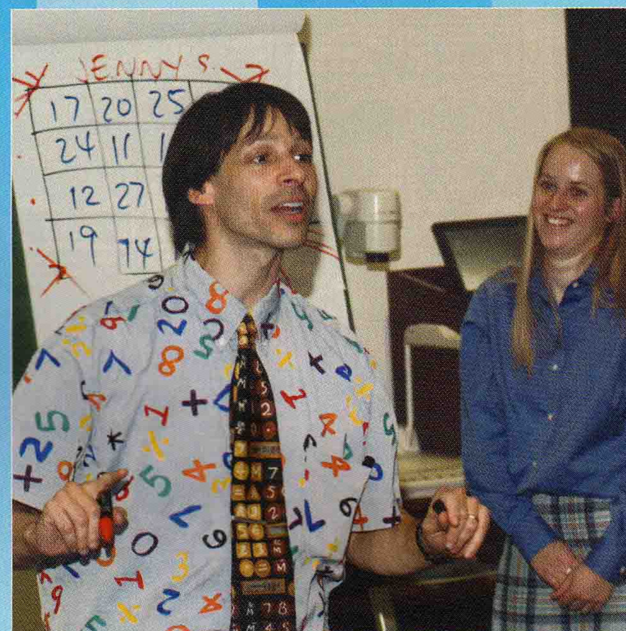
Double Birthday Magic Square



By Arthur Benjamin

The following magic square routine can be performed close-up or on stage. When I perform it close-up, I do it on the back of my business card. When I perform it onstage, as described here, I use a large pad mounted on an easel.

Display an empty 4 by 4 grid, and invite someone to give you a birthday that is special to her, including the year. Suppose Deena gives you the date March 19, 1961. You commemorate this date by writing the four numbers 3, 19, 6, 1 in the first row of the magic square. *"If we add the digits of your birthdate, we get 3 + 19 + 6 + 1 equals 29. This is your magic number."* Write the number 29 below the grid and explain, *"I will now attempt to fill out the rest of this grid in such a way that we reach 29 in as many ways as possible. This will only take a minute."* As I say this, I am writing numbers in the magic square in a random-looking order. When completed, it looks like this:

3	19	6	1
5	2	2	20
2	7	18	2
19	1	3	6

29

"We know that the first row sums to 29, would you choose any other row, number 2, 3, or 4." Say she indicates row three. Slowly, you say, *"Notice that 2 plus 7 is 9, plus 18 is 27, plus 2 is 29."* Circle the number 29 below the magic square, then continue with more speed. *"Adding the other rows we get..."* Point to row two: "...5, 7, 9, 29." Point to row four: *"19, 20, 23, 29. Now would you choose a column – one, two, three, or four?* Say she picks column four. *"Check this out. 1 plus 20 is 21, plus 2 is 23, plus 6 is 29. The other columns give you 3, 8, 10, 29; then 19, 21, 28, 29; and finally, 6, 8, 26, 29. How about that?"*

After the applause subsides, you continue: *"Now Deena, since this was your magic square, I decided, at no extra charge, to give you these diagonals as well. Check it out: 19 plus 7 is 26, plus 2 is 28, plus 1 is 29. The other diagonal is 3, 5, 23, 29. But I didn't stop there either, I decided that since this was your magic square, wouldn't it be great if we could get these four in the center to add up as well. Notice 2 plus 2 is four, plus 18 is 22, plus 7 is 29. But did I stop there? No. You may have noticed, I put a little extra attention in this corner."*

With a colored marker, I draw a box around the four numbers in the upper left corner. *"I did that so I could get these four numbers 3 plus 19 is 22, plus 2 is 24, plus 5 is 29. And I figured, heck, as long as we got that group of four, let's have a party. We may as well get this group of four..."* Boxes are drawn around the other three quadrants: *"...6, 7, 27, 29; then 2 9, 10, 29; and 18, 20, 26, 29! But did I stop there? No! I decided that Deena wouldn't be happy unless we got this group of four..."* A box is drawn around the top center numbers

19, 6, 2, 2: "…and this group of four," boxing the bottom center numbers 7, 18, 1, 3. *"Now I have to apologize to you Deena, I was not able to get this group of four nor that group of four to add up,"* indicating the left center numbers 5, 2, 2, 7 and the right center numbers 2, 20, 18, 2. *"But I had to do it that way, if I was going to be able to get these four in the corners to add up. I knew that would be important to you."* The four corner numbers are circled: *"3 plus 1 is 4, plus 6 is 10, plus 19 is 29."*

Pause for a moment, then say, *"But here's the cool part. Not only do the four corners add up to 29, but if you look at them closely, you'll notice that those corner numbers are 3, 19, 6, and 1. I was able to give you your birthday again. But don't take my word for any of these calculations, please keep this as a birthday souvenir, and let's give Deena a nice round of applause."*

The secret of the magic square you create is based on the following table:

A	B	C	D
C-1	D+1	A-1	B+1
D+1	C+1	B-1	A-1
B	A-2	D+2	C

A+B+C+D

Notice that every row, column, diagonal, and quadrant mentioned in the presentation — along with many, many others that I usually do not mention — adds to the magic number A + B + C + D. It is important to note that the center left and center right quadrants do not add up to A+ B+C+D.

You do not need to memorize this table — I haven't. In fact, once you write down the number C+1 in row three, column two (which I will denote as cell (3,2)) then the rest of the table is forced. (This idea was inspired by an article by Harry Lorayne that appeared in the April 2005 *Genii*, as well as in his book *Mathematical Wizardry*.) After writing the number C+1 in cell (3,2), I then write the number B in cell (4,1) putting us in this situation:

A	B	C	D
	C+1		
B			

Now since you know that the diagonal must sum to A+B+C+D, then that forces the (2,3) entry to be A-1. Thus, when creating the square, I mentally add the three diagonal numbers 19+7+1 = 27, and since the magic number is 29, that forces the missing number to be 2. Once the (2,3) entry is filled in, this will force the (2,4) entry to be B+1 (2 + 6 + 1 = 9 forces the (2,4) entry to be 20.) The top center quadrant forces the (2,2) entry and the upper left quadrant forces the (2,1) entry. We now have the following situation:

A	B	C	D
C-1	D+1	A-1	B+1
	C+1		
B			

Notice that in the final grid, the (3,1) entry (D+1) is always the same as the (2,2) entry, so you can simply fill that in with no computations needed. (Note: Be sure not to add the center left quadrant, since that doesn't sum to A+B+C+D.) Next, I use the four center squares to get the (3,3) entry, then the lower left quadrant to get the (4,2) entry, then the bottom center quadrant to get the (4,3) entry. Finally, as your brain is starting to lose interest in adding numbers, you can instantly fill in the (3,4) entry, which is always equal to the (2,3) entry (A-1), and the (4,4) entry is, by design, always equal to C, the (3,1) entry.

One drawback to this method is that the magic square created has some aesthetic flaws. For instance, the magic square created will always have at least 4 pairs of repeated numbers, and sometimes more. Worse than that, if the birthday is in January, then the (4,2) entry will be -1. (When I get a January date, then I will start entry (3,2) with C-1 instead of C+1. The resulting magic square has all of the same symmetries, but with all of the plusses and minuses reversed. This avoids the negative problem unless C = 0 or D = 0 or 1.) Also, I don't like the fact that when B is much larger than the other numbers, as in our example, the resulting magic square has four numbers that are conspicuously larger than the other twelve numbers. This problem of four large numbers is a blemish of many magic square methods.

These aesthetic problems do not exist in my other magic square routine, "Interactive Magic Squares," described in my new book, *Secrets of Mental Math*. I learned the idea behind this routine in the 1970's from Annemann's *Book Without a Name,* which was reprinted in the back of Tannen's 1975 catalog. In this routine, the volunteer selects any number over 40, then points to the squares in any order she likes, and you instantly enter the number. The resulting magic square has all of the same symmetries previously mentioned, but the numbers are all different, and they are practically consecutive.

On the other hand, what I like about the double birthday method for magic squares is that it is personalized, requires no setup, and has a strong finish and with a sense of closure, bringing it back to the volunteer's birthday. ❖

Arthur Benjamin is a professor of mathematics at Harvey Mudd College in Claremont, California, and his new book *Secrets of Mental Math* will be released by Random House this month.

Photo courtesy of American University by Hilary Schwab.