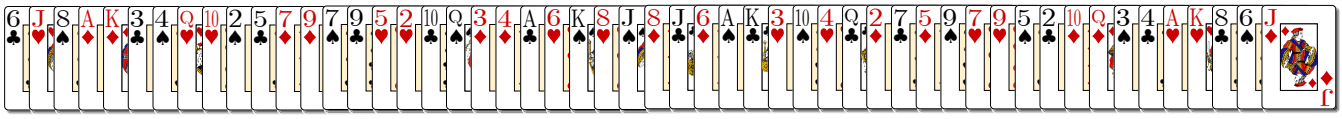
Seven-Card Stud Magic and the Gilbreath Principle



Arthur Benjamin

The magician approaches you with a deck of cards to perform a psychic experiment, involving a game of poker, where the magician never touches the cards. To begin, you are allowed to cut the deck as often as you'd like. Then you deal about half the cards on to the table and give the two piles a riffle shuffle. Next, as in the game of seven-card stud, you deal 14 cards alternately between the magician and yourself, where the first four cards are dealt face down and the remaining 10 cards are face up. Now for the magical part. Even though the magician hasn't touched the cards throughout the experiment, after looking at their face down cards (as in figure 3), and a little bit of dramatic concentration, they can reveal your face down cards. How can they do it?

In my opinion, this is one of those tricks where the secret is just as interesting as the trick itself. Although the magician never touches the cards throughout most of the trick, the deck is arranged in advance in the special way depicted in figure 1. (Magicians say that the deck is stacked.)

The first 13 cards are:

8C KH 3S 10D 2C 7H 9S 5D QC 4H AS 6D JC

and the card values repeat cyclically. Magicians call this the "8 Kings" stack, and the values can be memorized through the following nursery rhyme:

Eight kings threatened to save
95 ladies for one sick knave.

Notice that the suits also repeat in a cyclic order: Club-Heart-Spade-Diamond, known to magicians as the CHaSeD order. Because the 13th card is a club, the next 13 cards will begin and end with a heart, and the pattern continues, beginning with the eight of hearts followed by the king of spades. The first thing to notice about this stack is that when you cut the cards at the beginning of the trick, the resulting deck retains its cyclic order. Every set of four consecutive cards contains a club, heart, spade and diamond (in that circular order) and every collection of 13 consecutive cards contains all card values (in the 8 Kings circular order). After dealing some cards to form two piles and riffling those two piles together, some of that structure is destroyed, but the cards still maintain a rather surprising structure due to the *Gilbreath principle*.

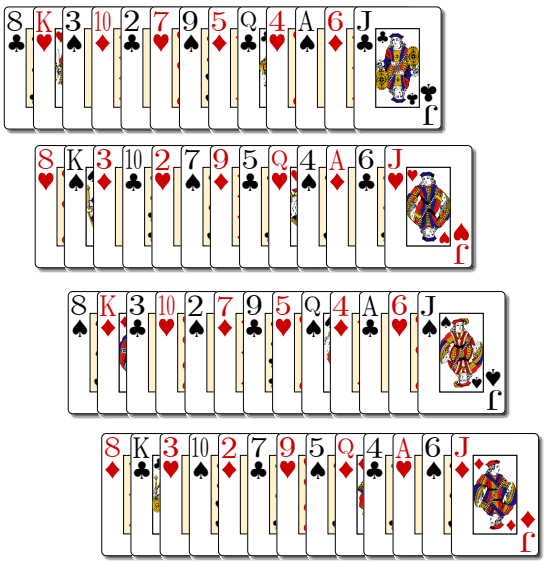


Figure 1. The 8 Kings stack.

The Gilbreath principle, discovered by professional engineer and amateur magician Norman Gilbreath in the 1960s, applies to any deck of N cards that is stacked with a cyclic structure of d cards that repeats (N/d times) throughout the deck. For example, the 8 Kings stack has $N = 52$ cards and has many cyclic structures: for $d = 2$, we see that the colors alternate (red and black); with $d = 4$, we have the club-heart-spade-diamond cycle; and with $d = 13$, we have the 8-K-3-10-2-7-9-5-Q-4-A-6-J cycle.

The Gilbreath principle implies that after dealing any number of cards to create two piles, followed by a riffle shuffle, the top collection of d cards will still have one card of each type. (For example, the top four cards will have one card of each suit, and the top 13 cards will have one card of each value.) See figure 2 for an example. Moreover, this property also holds for the next set of d cards, and the next set of d cards, …, going all the way to the bottom set of d cards. For the purposes of our magic trick, this tells us that the first four cards are guaranteed to be a club, heart, spade, and diamond (in some order) and the first 13 cards are guaranteed to consist of all the values Ace through King (in some order).

[see title area for full width image]
Figure 2. Starting with the 8 Kings stack, dealing 25 cards, and doing a riffle shuffle, this is a possible ordering.

You can find a proof and generalizations of the Gilbreath principle in many publications, but here is an informal *proof by example* that gives the basic idea. Suppose we start with a 52-card deck where the cards are arranged in ascending ace through king order, repeated four times with an ace on top and a king on the bottom. Then suppose we deal 21 cards from the top of the deck so that pile A has 21 cards with an ace on the bottom and an eight on the top. Pile B has 31 cards with a nine on the top and a king on the bottom. Next, we riffle shuffle pile A into pile B, and let's suppose that 10 of the top 13 cards come from pile A. Necessarily these 10 cards would be 8, 7, 6, 5, 4, 3, 2, A, K, Q, and the 3 cards that from pile B would necessarily be 9, 10, J. Hence after a riffle shuffle, the top 13 cards must be the ace through king in *some* order. If we mentally remove these 13 cards, we still have an ace on the bottom of pile A, and a king on the bottom of pile B, but now there is a Jack on top of pile A, which complements the Queen that is now on top of pile B. Thus, we essentially have the same setup as before, where we start with 39 cards with ace through king in ascending order, and we have just dealt 11 cards on to the table, so the same logic applies to the smaller problem. If you're still not convinced, pick up a deck and try it yourself!

Let's exploit the Gilbreath principle to explain the *Seven-Card Stud Mystery*. I'll use the patter that I use when performing it (in italics) and describe the secret stuff as we go along.

In the game of seven card stud, both players are dealt five cards face up and two cards face down. And if you can figure out the other players down cards, you have a big advantage. Let's play a hand. To begin, go ahead and cut these cards as often as you'd like. To mix things up further, deal about half the cards, one at a time, onto the table, but it doesn't have to be exactly half the cards. I'll turn away so I can't see how many cards you deal. Now go ahead and riffle shuffle the two piles together, so that the cards are well-mixed. Next deal the cards, one at a time, starting with a face down card for me, a face down card for you, a face down card for me, and a face down card for you. Then deal ten more cards but deal these cards face up.

Go ahead and look at your face down cards, and I will look at mine. Do you have any idea what my face down cards could be? No?

I turn over my face down cards, say they are the six of clubs and the ace of diamonds (figure 3).

Figure 3. We can use the Gilbreath principle on this collection of 14 dealt cards to determine the remaining two face-down cards.

By the Gilbreath principle, I know that the first four dealt cards have one card of each suit. As a result, because my cards are a club and a diamond, the face down cards must be a heart and a spade. But I don't say that right away.

I will try to read your mind to determine your face down cards. Let me start with the colors. I'm not getting a strong impression of either color, so I think you have one card of each color. Is that right? Good! Okay, let me focus on the suits. It's getting a little clearer. I'm pretty sure that the red card is a heart and the black card is a spade. Right? Good!

While revealing the suits, I'm looking at the face up cards to apply the Gilbreath principle again. We know that the first 13 cards dealt must consist of all 13 values. So I look at all of the face up cards (including the cards that were originally my face down cards) except for the 14th card (which is their last card—the 10 of clubs in this example). The remaining 11 cards will contain 11 different values. For instance, in the deal in figure 3, we see all the possible card values except for eight and jack. Consequently, we know that their face down cards must be an eight and a jack. And because we know the suits, we almost know their cards completely. The only problem is that we don't know if they have the eight of hearts and jack of spades, or if they have the eight of spades and the jack of hearts.

Okay, so I know that you have a heart and a spade. Let me try to visualize them. I'm seeing a picture card, but I don't think you have two picture cards. So one of your cards is a picture card and one is a number card, but which one is the heart? I could be wrong on this, but is the picture card a heart? Good! Okay, then I think I've got it. Your cards are the jack of hearts and the eight of spades. Right? Thank you very much!

Your guess of the heart card has at least a 50% chance of being right, but if you look at the surrounding cards, particularly the cards that were previously face down, you can guess right almost all the time. For instance, I see among the face-up cards 4-A-6 ("for one sick") in figure 3. From this, I expect the Jack to be the card that follows the six of clubs, which is the jack of hearts, using the CHaSeD ordering. Thus, we can be quite confident that the two face-down cards are the jack of hearts and the eight of spades.

Now it's your turn as the magician. I have dealt the 14 cards as shown in figure 4. Can you determine the two face-down cards? (The answer is hidden somewhere on this page).

Figure 4. What are the two face-down cards?

Further Reading

Matt Baker, "The Gilbreath Principle," *www.vanishingincmagic.com/blog/the-gilbreath-principle*, Oct. 14, 2020.

Colm Mulcahy, Card Colm, The First Norman Invasion, *www.maa.org/community/maa-columns/past-columns-card-colm/the-first-norman-invasion*, August 2005.

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Colm Mulcahy, *Mathematical Card Magic: 52 New Effects*, CRC Press, 2013.

Persi Diaconis and Ron Graham, *Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks*, Princeton University Press, 2011.

Art Benjamin, *Math and Magic*, DVD course from The Great Courses, 2018.

***Arthur Benjamin** is the Smallwood Family Professor of Mathematics at Harvey Mudd College and also a professional magician. He has performed his math and magic to audiences all over the world, including on the TED stage. He is a past editor of* Math Horizons *and was recently inducted into the American Backgammon Hall of*

Fame. He extends nine hearts and two diamonds to his family for their constant support.