This thesis will investigate the advection and diffusion of a passive scalar in helical pipes. By assuming that the curvature and torsion are small (equivalent to small Dean number) and the Reynolds number is moderate, one can use a closed form approximation, due to Dean (1927) and Germano (1982), for the induced recirculation. Using a simulation developed this summer, the problem will be investigated numerically with a range of localized initial conditions.

Already, we have found that the problem is governed by two parameters: a nondimensional diffusion constant $D$ (typically small), and the scaled ratio of torsion to curvature $\lambda$. At small times, the longitudinal width of the particle distribution, $\sigma$, is governed by diffusive effects ($\sigma \propto \sqrt{Dt}$). At large times, Taylor diffusion dominates ($\sigma \propto \sqrt{t/D}$). However, at intermediate times, a ballistic region exists where the width spreads linearly, as postulated by Mezic & Wiggins (1994).

This project has two parts. First, we will investigate numerical solutions to provide insight into the behavior of the helical pipe flow. Second, we will analyze the system of PDEs generating the flow to explain the various width-scaling regions and to find how the various diffusion regimes scale with $D$ and $\lambda$.

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References:


Relevant Course Work:

Math 136, 180, 182
Computer Science 60, 70