

# Procedural Support for Cooperative Negotiations

## Theoretical Design and Practical Implementation

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**Abstract:** We discuss the theoretical design of algorithms for solving distributional conflicts within groups. We consider an algorithm to be *procedural* if the implementation of the outcome requires the participation of the players, or if it can even be conducted by the players themselves without computational assistance. We compare two procedures for multilateral problems of fair division; both establish envy-freeness, given the possibility of monetary compensations between players.

## 1 Introduction

Negotiation is always related to distribution — either of benefits or of costs. Since this distributive dimension is present in every bargaining situation, negotiating parties must inevitably deal with conflict in one way or another. *Negotiation analysis* is a practically oriented field of research that seeks to offer negotiators *prescriptive* advice on how to establish a desirable group decision in practice. A major goal is the development of procedures for dealing with conflicts in a constructive manner. Since negotiation always involves the division of some “pie,” a promising approach seems to be the analysis of problems of fair division.

The theoretical development of fair-division procedures has become quite popular over the past decade. Of particular interest for group decisions are procedures dealing with *multilateral* distribution problems. New theoretical results show conditions for the existence of envy-free solutions, where an outcome is considered to be *envy free* if each party believes that its share is at least as desirable as that of any other party. The mere proof of existence, however, does not yet provide the desired outcome. More helpful in this respect are constructive proofs of envy-freeness, yielding solutions via algorithms that are implementable, e.g., as a computer algorithm.

In this paper we extend the notion of a constructive proof: we consider an algorithm to be *procedural* if the implementation of the group decision involves the active participation of the players, or if it can even be conducted by the players themselves without computational assistance. This aspect seems to be essential for the acceptance of analytical support, because, as Spector (1997) observes, practitioners are often reluctant to use support tools that offer solutions simply at the push of a button. What is required in his view is (procedural) assistance that guides negotiators to a mutually acceptable agreement.

## 2 A Multilateral Fair-Division Problem

Consider the following situation: Four individuals wish to rent a house with four bedrooms for a total rent of, say \$100. Is there a possibility of allocating rooms to players and dividing the total rent between rooms in such a way that no individual will wish to trade rooms with another? Under relatively mild assumptions concerning players' preferences such an *envy-free* allocation exists — the question is how to find it.

Suppose that each player  $P_i$ ,  $i=1, \dots, 4$ , is able to express her valuations of the individual rooms  $R_j$ ,  $j=1, \dots, 4$ , via monetary bids, as given in Table 1 (cf. Haake, Raith, and Su, 1999).

	R1	R2	R3	R4
P1	50	20	10	20
P2	60	40	15	10
P3	0	40	25	35
P4	50	35	10	30

Table 1: Players' monetary valuations (bids) of rooms

In addition, if players' preferences over money are linear, then each player  $P_i$  will prefer the room  $R_j$  which offers her the maximal discount, given by the difference between her individual bid,  $\dots$ , and the actual price of the room.

## 3 An Interactive Choice Procedure

A possible procedure for allocating rooms and splitting the rent is to assign prices to the individual rooms that sum to the total rent, and then let each player choose the room she likes best. For example, if all rooms are priced equally at \$25, then P1, P2, and P4 will all wish to have R1, while P3 would choose R2. With only one player allowed in each room, this price distribution is sure to create envy. However, if a pricing scheme can be found such that each room is desired by a different player, then the allocation will be envy-free.

The interactive fair-division procedure developed by Su (1999) is based on a constructive proof of the combinatorial lemma given by Sperner (1928), reformulated to find an envy-free pricing scheme. The algorithm sequentially offers each player a specific pricing scheme and then lets this player choose her favorite alternative. For example if P1 is offered the pricing scheme  $[0,0,0,100]$ , her best choice will be R1, because this maximizes her discount. Under the Sperner approach, each player's choice of an alternative determines which pricing scheme is offered next and to which player. The interactive procedure consistently works its way towards envy-freeness, and it provably converges on an envy-free pricing scheme for any given level of precision with which players can determine the value of rooms. For our example above, the pricing scheme  $[45,25,10,20]$  induces P1, P2, P3, and P4 to choose R1, R2, R3, and R4, respectively. Subtracting room prices from players' valuations, given

in Table 1, shows that these choices give players maximum discounts for the given pricing scheme.

Implemented on a computer, the Sperner choice procedure operates in the form of an artificial mediator. The mediation procedure can be employed using the web-based “Fair-Division Calculator” (<http://www.math.hmc.edu/~su/fairdivision/calc/>). Parties can actively take part in the process and are allowed to make choices to their individual benefit. The Fair Division Calculator plays the part of the mediator by offering pricing schemes in a manner that will lead the group to an envy-free allocation, provided that participants’ choices are consistent and truthful. If a player behaves inconsistently, either by accident or due to a change of preferences, the mediator can react by pointing to the inconsistency or by accepting the modified preferences, thus ignoring previous (inconsistent) choices. Strategic choices, on the other hand, cannot be unmasked as long as they are consistent. However, players will find it quite difficult to manipulate the procedure, because the other players’ preferences are never fully revealed. Indeed, if the choices of the other players are kept confidential up to the final round, then a player, when offered a pricing scheme, will never know whether the next choice will be her last.

Viewed as a mediation process, the main difficulty with the Sperner choice procedure is that the operations of the mediator are largely incomprehensible to the group. Although each player has a free choice of alternatives and thereby determines the direction of the process, she will generally have a problem understanding the logic behind the evolution of pricing schemes. Players must completely rely on the artificial mediator to arrive at an envy-free outcome. Without this trust, the choice procedure may not be acceptable for the group.

## 4 A Mediating Compensation Procedure

Acceptability of a live mediation procedure typically requires additional features that relate the process to the players involved. To be acceptable, a mediation procedure should be *intuitive*, meaning that the implementation of an outcome must accord to the players’ way of reasoning. In addition, the procedure should be *plausible*, meaning that the individual steps must match the players’ way of arguing. And finally, the procedure should be *manageable*, meaning that the required calculations must correspond to the players’ computational abilities. With the compensation procedure developed by Haake, Raith, and Su (1999), players are able to see how the outcome evolves through the interaction of their preferences. They can, therefore, understand the mediation process, and even conduct it themselves.

The compensation procedure is initialized by having players individually *and* simultaneously submit monetary bids for the individual rooms. All bids, disclosed together, then form the basis of the mediation process leading to envy-freeness.

The first step of the mediation procedure is to assign rooms to players, such that the sum of players’ bids is maximized. In our example, this *utilitarian* allocation is achieved by assigning R1, R2, R3, and R4 to players P1, P2,P3, and P4, respectively (in Table 1, the diagonal assignment). The utilitarian assignment ensures that no sub-group of players can mutually gain by trading rooms. Consequently, there can be no two-way or cyclical

envy, since this could be eliminated via a mutually beneficial trade. However, a utilitarian assignment may still leave individual players envious. As Table 1 shows, P1 is satisfied with R1, but P2 and P4 prefer R1 as well, while P3 would rather have R2. As any envious player can verify, though, no player is willing to enter a trade with her.

The second step of the procedure is to compensate all envious players by an amount sufficient to eliminate their envy. The most plausible starting point is to begin with those players whose maximum envy is directed towards a non-envious player. A non-envious player always exists, because if all players were envious, there would be efficient trading opportunities. As Table 1 shows, P1 is the non-envious player, and P2 and P4 need to be compensated for their envy; P3 envies an envious player and can, therefore, be taken care of in a later round. By adding \$20 to the value of R2 and R4, P2 and P4 are not envious of P1 anymore. Indeed, P2 is now envy-free, but, as the left-hand panel of Table 2 shows, P4 now envies P2, and P3 is even more envious of P2 than before. In the second round P3 and P4 are, therefore, compensated by adding \$35 to R3 and an additional \$5 to R4. As the right-hand panel of Table 2 shows, all players are now satisfied with their assigned rooms.

	R1	R2	R3	R4
P1	50	40	10	40
P2	60	60	15	30
P3	0	60	25	50
P4	50	55	10	50
comp.	0	20	0	20

	R1	R2	R3	R4
P1	50	40	45	45
P2	60	60	50	35
P3	0	60	60	55
P4	50	55	45	55
comp.	0	20	35	25

Table 2: Players' most preferred rooms after one and two rounds of compensations

As Haake, Raith, and Su (1999) formally show, after  $k$  rounds of compensation at least  $k+1$  players will be envy-free. Envy-freeness for 4 players thus requires at most 3 compensation rounds; in our example only two are necessary. Note that, even if players do not understand the formal proof of this claim, they can verify its validity at every step of the procedure.

After envy-freeness is achieved, the group must bear the cost of compensations, \$80, plus the rent for the house, \$100. If the total cost is shared equally, each player pays \$45. Net of compensations, the final envy-free prices for the rooms are [45,25,10,20].

Although the compensation procedure can generally be conducted without computational assistance, larger groups may wish to have precise guidance as the mediation process becomes longer. Moreover, the utilitarian assignment becomes increasingly difficult to find without analytical support. Implemented on a computer, the compensation procedure provides the complete mediation process, based only on players' initial valuations of rooms — the compensation procedure can be employed using the Fair Division Calculator (FDC). However, in contrast to the Sperner choice procedure in which the FDC works its way to envy-freeness by interacting with the players, the FDC does not operate as a mediator under the compensation procedure. Instead, it supplies the mediation process that can be used by a mediator or the group itself as a map to envy-freeness.

## 5 Decentralized Procedural Support

Adapting group interaction to the support environment is a common feature of many computerized support systems. However, if the system imposes too many restrictions on the group decision process, users may find it too artificial and, therefore, be reluctant to use it in practice. Practitioners often find their flexibility constrained by the way they are forced to interact with the system. The challenge in designing a practicable support system, therefore, lies in the development of analytical techniques that complement the group's interaction in a "natural" way.

As a web-based application the Fair Division Calculator can be employed from any remote location. Equipped with a variety of different mediation procedures, the FDC provides full analytical mediation support. Depending on the amount of information that parties are willing to reveal, the FDC can assist by offering the group a workable mediation process, or it can be part of the process and operate as an artificial mediator.

Since the FDC offers group support via a single user interface, decentralized group interaction requires additional support facilities, allowing parties to interact with each other from remote locations. This has been accomplished by integrating the FDC into a negotiation platform under the the internet system ARTUS (<http://artus.wiwi.uni-bielefeld.de/>). ARTUS provides an infrastructure for group decision making over the Internet in the form of virtual tables. The system includes options for defining the method and structure of multilateral interaction, and it records the complete interaction for security, documentation, and evaluation (for a detailed documentation see Raith and Wilker, 1998).

Due to its accessibility via the internet and its variable user interfaces, ARTUS can be adapted to many different forms of decentralized group interaction, with or without analytical support. A mediation process via ARTUS allows the participation of a natural mediator, equipped with all the psychological skills necessary for guiding parties in a conflict situation, and assisted by the analytical skills of a support system such as the Fair Division Calculator. Group support can thus be integrated naturally in an artificial environment.

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