As usual, “R” means read, but do not do the problem (do it in your head).

1. Let $K$ be an infinite field, and $f$ and $g$ in $K[x_1, ... , x_n]$. Show that $f = g$ as polynomials if and only if $f, g : \mathbb{A} \to K$ are the same function.

2. If $W = \mathcal{Z}(f_1, ..., f_s)$ and $V = \mathcal{Z}(g_1, ..., g_t)$ are algebraic sets in $\mathbb{A}^n$, show that $W \cup V$ and $W \cap V$ are algebraic sets.

R3. Show that every finite subset of $\mathbb{A}^n$ is an algebraic set.

4. Show that $X = \{(x, x) : x \in \mathbb{R}, x \neq 1\}$ is not an algebraic set in $\mathbb{R}^2$. Hint: if $f \in \mathbb{R}[x, y]$ vanishes on $X$, what can be said about $f(1, 1)$?

5. Let $V$ be an algebraic set in $\mathbb{A}^n$ (over a field $K$). Let $\mathcal{I}(V)$ be the ideal of all polynomials in $K[x_1, ... , x_n]$ that vanish (evaluate to 0) on the set $V$.

Call an ideal $I$ in a commutative ring a radical ideal if for all $r \in R$, $r^m \in I$ implies $r \in I$.

Show that $\mathcal{I}(V)$ is a radical ideal.

6. For any nonzero polynomials $f$ and $g$, show that:
   \begin{itemize}
   \item[(a)] $LT(fg) = LT(f)LT(g)$ and $\partial(fg) = \partial(f) + \partial(g)$.
   \item[(b)] $\partial(f + g) \leq \max\{\partial(f), \partial(g)\}$ with equality if $\partial(f) \neq \partial(g)$ (where $\leq$ is a monomial order).
   \end{itemize}