Math 131 — Homework 3

READ Problem D. Let u be an upper bound of non-empty set A in $\mathbb{R}$. Prove that u is the supremum of A if and only if for all $\epsilon > 0$ there is an $a \in A$ such that $u - \epsilon < a$.

Note that to show that "S if and only if T" you must show that S implies T, and T implies S.)

READ Problem E. Let A, B be nonempty subsets of $\mathbb{R}$ that are bounded above, and let $A + B = \{a + b : a \in A, b \in B\}$. Show that

$$\sup(A + B) = \sup A + \sup B.$$

Problem F. Let A, B be nonempty subsets *of positive real numbers* that are bounded above, and let $A \cdot B = \{ab : a \in A, b \in B\}$. Show that

$$\sup(A \cdot B) = \sup A \sup B.$$

Problem G. (a) Let A be a nonempty subset of $\mathbb{R}$ and suppose that $s = \sup A$ belongs to A. If b is not in A, show that $\sup(A \cup \{b\})$ is equal to the larger of the two numbers s and b.

(b) Use this to show that a nonempty finite set A contains its supremum. [Hint— use induction: show it is true first for a one-element set, then show that *if* it is true for an n-element set then it must be true for an $(n + 1)$-element set.]

Do also **Chapter 1 (6ab, 6cd, 12, R13, 15)**.

Comment: When you are asked a question, e.g., problem 1.15, you should always give justification.