

MATH 172 (SU): HW NUMBER 10

As usual, “R” means read, but do not do the problem (do it in your head).

1. Let \mathbf{K} be an infinite field, and f and g in $\mathbf{K}[\mathbf{x}_1, \dots, \mathbf{x}_n]$. Show that $f = g$ as polynomials if and only if $f, g : \mathbb{A}^n \rightarrow K$ are the same function.

2. If $W = \mathcal{Z}(f_1, \dots, f_s)$ and $V = \mathcal{Z}(g_1, \dots, g_t)$ are algebraic sets in \mathbb{A}^n , show that $W \cup V$ and $W \cap V$ are algebraic sets.

R3. Show that every finite subset of \mathbb{A}^n is an algebraic set.

4. Show that $X = \{(x, x) : x \in \mathbb{R}, x \neq 1\}$ is *not* an algebraic set in \mathbb{R}^2 . Hint: if $f \in \mathbb{R}[x, y]$ vanishes on X , what can be said about $f(1, 1)$?

5. Let V be an algebraic set in \mathbb{A}^n (over a field K). Let $\mathcal{I}(V)$ be the ideal of all polynomials in $\mathbf{K}[\mathbf{x}_1, \dots, \mathbf{x}_n]$ that vanish (evaluate to 0) on the set V .

Call an ideal I in a commutative ring a *radical ideal* if for all $r \in R$, $r^m \in I$ implies $r \in I$.

Show that $\mathcal{I}(V)$ is a radical ideal.

6. For any nonzero polynomials f and g , show that:

(a) $LT(fg) = LT(f)LT(g)$ and $\partial(fg) = \partial(f) + \partial(g)$.

(b) $\partial(f + g) \leq \max\{\partial(f), \partial(g)\}$ with equality if $\partial(f) \neq \partial(g)$ (where \leq is a monomial order).