Complex Arithmetic

Due date: Tuesday, January 29th in class.

A1: Simplify each of the following as much as possible. (In other words, write each in the form $a + bi$.)

(a) $i^{2019} + i^{-2019}$

(b) $(1 + 2i)^3$

(c) $\dfrac{1}{1 + 2i}$

(d) $\dfrac{1 + i}{1 - i}$

(e) $\dfrac{3}{i} + \dfrac{i}{3}$

(f) $\text{Im}\left(\dfrac{a + bi}{c + di}\right)$ (assume a, b, c and d are real)

(g) $\left|\overline{(1 - 2i)^3} - (1 + i)^3\right|$

Note: To save some effort, think about whether $\overline{z_1 z_2}$ is the same as $\overline{z_1}\,\overline{z_2}$ for any two complex numbers z_1 and z_2.

A2: (a) What is the graphical relationship between z and its multiplicative inverse, z^{-1}, if $|z| = 1$?

(b) What is the graphical relationship between z and $-z$?

(c) What is the graphical relationship between z and $-iz$?

(d) What is the graphical relationship between z_1, z_2, and $(z_1 + z_2)/2$?

(e) Describe the set of all points in the complex plane that satisfy $z = \overline{z}$.

(f) Describe the set of all points in the complex plane that satisfy $1 \leq |z| < 2$.

A3: What are all possible solutions of $z^4 + 4 = 0$? From this information, write out a complete factorization of $z^4 + 4$.

A4: Compute all possible values of $(i - 1)^{1/3}$.

A5: We introduced the polar form for complex numbers $a + bi = re^{i\theta}$ where **Euler's Formula**

$$e^{i\theta} \equiv \cos\theta + i\sin\theta$$

introduces the exponential notation for describing the polar angle θ. Fortunately, this exponential function obeys all the rules we learned for the real exponential function, but strictly speaking we should prove them all. Prove the product rule

$$e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1 + \theta_2)}.$$

You will need to remember your trigonometric identities for $\cos(\theta_1 + \theta_2)$ and $\sin(\theta_1 + \theta_2)$.