Research Area: Differential Geometry

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The research field has been developped naturally from my mathematical training both in differential geometry and topology at UPenn. Over the years, I have broadened it, have made the connection and applications to string theory in theoretical physics, and have involved many outstanding undergraduates at Claremont in the research.

After I published the main results from my thesis "The Stable 4-dimensional Geometry of the Real Grassmann Manifolds" in the *Duke Mathematical Journal*, I tried to prove the uniqueness theorem involved. With the support of Robert Bryant, I had a chance to take a short course at MSRI on "Exterior Differential Systems" and wrote a paper with Dan Grossman on "Uniqueness of Volume-minimizing Submanifolds Calibrated by the First Pontryagin Form," published in the *Transactions of the AMS*.

I then extended techniques that made the method of calibrations an effective tool for identifying volume-minimizing cycles on Grassmann manifolds now applicable on Calabi-Yau 4-folds. One direction is the application of techniques from my previous work to identify Cayley cycles in the Calabi-Yau 4-fold $T^4_{\mathbf{C}}$, a problem suggested to me by Professor Gang Tian. These Cayley cycles will provide explicit examples in our current search for new holomorphic invariants for Calabi-Yau 4-folds and for the further development of mirror symmetry. I have carried out some work on this together with my undergraduate student Christopher Pries. Our paper on "Examples of Cayley Manifolds in $\mathbf{R}^8$" has been published in the *Houston Journal of Mathematics*. A second paper on "Classification of Cayley Graphs" has been submitted to the *Duke Mathematical Journal*. A paper on "Cayley Cycles in T^8" is in preparation.

The next research work uses the techniques obtained in the work on Cayley cycles to identify associative cycles in the G_2 holonomy manifold $\mathbf{R}^6 \times S^1$, a problem proposed to me by Professor Edward Witten. Such cycles, which are candidates for representations of certain fundamental particles, will provide explicit examples in Witten's M-theory, which is trying to unify the five known string theories. I have involved several of my best undergraduate students in this research. My paper with Ian Weiner on " Associative Manifolds Invariant Under 1-Parameter Subgroups of G_2" has been accepted by the *International Journal of Pure and Applied Mathematics*. Matt Holden, who wrote his Pomona College senior thesis with me, and I have submitted a paper on volume minimizing cycles

in $\mathbf{R}^6 \times S^1 \cong \mathbf{R}^7/\mathbf{Z}$. Currently, Ruben Arenas and I, are preparing a paper on "A new basis for Lie Algebra of G_2".

In addition, I have been generalizing my previous calibration techniques in identifying volume minimizing cycles in the real Grassmann manifolds to the case of complex Grassmann manifolds. I am (in collaboration with Dan Grossman) preparing a paper on "The 4-Dimensional Calibrated Geometry of Complex Grassmannians." We use the second Chern form and its related invariant forms as calibrations. Some of these volume minimizing cycles may provide specific examples in verifying the most general and important compactification results proved by Tian in his paper "Gauge Theory and Calibrated Geometry, I", of the moduli space of Yang-Mills connections defined on a vector bundle with structure group a compact Lie group.

Branching out from these, I have published other papers such as those with Z. Shen on "Levy Concentration of Metric Measure Manifolds" and with S. Jiang on "Knotty Matrix in Knots".

I have also conducted research on "Volume-preserving Great Circle Flows on the 3-sphere" by utilizing the structure of Grassmann manifolds in a collaboration with H. Gluck. This work has been published by *Geometriae Dedicata*.