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Math 61 Section 2
HW #3
due 5-20-04

$\frac{98}{100}$

4.3(1, 2, 8, 19, 21, 23, 29) 4.4(6(a), 6(c) only the part reffering to (a))

4.3 1. Find the point on the plane $2x-3y-z=4$ that is closest to the origin

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(a) using method 1

$f(x,y)=\sqrt{x^2+y^2+z^2}$ $z=2x-3y-4$

$f(x,y)=\sqrt{x^2+y^2+(2x-3y-4)^2}$

$$\nabla f(x,y)=\left(\frac{\frac{1}{2}(2x+4(2x-3y-4))}{\sqrt{x^2+y^2+(2x-3y-4)^2}},\ \frac{\frac{1}{\cancel{2}}(\cancel{2}y+\cancel{2}(2x-3y-4)(-3))}{\sqrt{x^2+y^2+(2x-3y-4)^2}}\right)=\vec{0}$$

$x+4x-6y-8=0$ $y-6x+9y+12=0$

$5x-6y-8=0$ $10y-6x+12=0$

$x=\frac{5y+6}{3}$

$5\left(\frac{5y+6}{3}\right)-6y=8$

$y=-\frac{6}{7}$ $x=\frac{4}{7}$ critical pt at $\left(\frac{4}{7},-\frac{6}{7}\right)$

Because the max is an unbounded situation, this critical point must be the minimum.

$z=2\left(\frac{4}{7}\right)-3\left(-\frac{6}{7}\right)-4$

$z=-\frac{2}{7}$

Closest point: $\left(\frac{4}{7},-\frac{6}{7},-\frac{2}{7}\right)$ ✓

b. using the Lagrange multipliers

$F(x,y,z,\mu)=f(x,y,z)+\mu(g(x,y,z)-4)$

$\frac{\partial F}{\partial x}=2x+2\mu=0$ $-x=\mu$ $-x=\frac{2}{3}y=2z$

$\frac{\partial F}{\partial y}=2y-3\mu=0$ $\frac{2}{3}y=\mu$

$2x-3\left(-\frac{3}{2}x\right)-\left(-\frac{1}{2}x\right)=4$

$\frac{\partial F}{\partial z}=2z-\mu=0$ $2z=\mu$

$2x+5x=4$

$7x=4$

$\frac{\partial F}{\partial \mu}=0=2x-3y-z-4$

$x=\frac{4}{7}$

$y=-\frac{3}{\cancel{2}}\times\frac{\cancel{2}}{7}=-\frac{6}{7}$

$z=-\frac{1}{\cancel{2}}\times\frac{\cancel{2}}{7}=-\frac{2}{7}$

$\boxed{\text{Closest pt. at } \left(\frac{4}{7},-\frac{6}{7},-\frac{2}{7}\right)}$ ✓

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Use Lagrange multi to find critical points

2. $f(x,y) = y \quad 2x^2 + y^2 = 4$

$g(x,y) = 2x^2 + y^2 - 4$

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$F(x,y,\mu) = f(x,y) + \mu(g(x,y))$

$F(x,y,\mu) = y + \mu(2x^2+y^2-4)$

$F_x = 4\mu x = 0 \qquad x = 0$

$F_y = 1 + 2\mu y = 0 \qquad -\frac{1}{2y} = \mu \qquad 4\left(-\frac{1}{2y}\right)x = 0$

$\frac{x}{y} = 0$

$F_\mu = 2x^2+y^2-4 = 0$

$\cancel{2x^2}^{=0} \quad y^2 - 4 = 0$

$y = \pm 2$

critical pts. $(0,2), (0,-2)$ ✓

8 $f(x,y,z) = x+y+z \quad g_1(x,y) = y^2-x^2-1 \quad g_2(x,z) = x+2z-1$

$F(x,y,z) = f(x,y,z) + \mu_1(g_1(x,y)) + \mu_2(g_2(x,z))$

$F(x,y,z) = x+y+z + \mu_1(y^2-x^2-1) + \mu_2(x+2z-1)$

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$F_x = 1 - 2\mu_1 x + \mu_2 = 0 \qquad \mu_1 = \frac{1}{4x}$

$F_y = 1 + 2\mu_1 y = 0 \qquad \mu_1 = -\frac{1}{2y}$

$F_z = 1 + 2\mu_2 = 0$

$\mu_2 = -\frac{1}{2}$

$F_{\mu_1} = y^2 - x^2 - 1 = 0 \qquad \frac{1}{4x} = \frac{-1}{2y}$

$F_{\mu_2} = x + 2z - 1 = 0 \qquad y = -2x$

$(-2x)^2 - x^2 = 1$

$3x^2 = 1$

$x = \pm\frac{\sqrt{3}}{3}, \quad y = \mp\frac{2\sqrt{3}}{3}$

$z = \frac{1-x}{2} = \frac{1 - \pm\frac{\sqrt{3}}{3}}{2}$

$z = \frac{1}{2} \mp \frac{\sqrt{3}}{6}$

critical pts. at $\left(\frac{\sqrt{3}}{3}, -\frac{2\sqrt{3}}{3}, \frac{1}{2} - \frac{\sqrt{3}}{6}\right), \left(-\frac{\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}, \frac{1}{2} + \frac{\sqrt{3}}{6}\right)$ ✓

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19. Find the maximum and min values of $f(x,y) = x^2 + xy + y^2$ on the closed disk $D = \{(x,y) \mid x^2+y^2 \le 4\}$

$f_x = 2x + y = 0$
$f_y = x + 2y = 0 \qquad x = -2y$
$2x - \frac{1}{2}x = 0$
$\frac{3}{2}x = 0$
$x = 0 \qquad$ critical pt at $(0,0)$ ✓
$y = 0$

$f_{xx} = 2 > 0$
$f_{xy} = 1 \qquad D = f_{xx} f_{yy} - (f_{xy})^2 = 2(2) - 1 = 3 > 0$
$f_{yy} = 2$

$f(0,0) = 0$

$\boxed{\text{So } (0,0) \text{ is a minimum}} \Rightarrow \boxed{f_{min} = 0}$ ✓

$F(x,y,\mu) = f(x,y) + \mu(x^2+y^2-4)$
$F(x,y,\mu) = x^2+xy+y^2 + \mu(x^2+y^2-4)$

$F_x = 2x + y + 2\mu x = 0 \qquad \mu = -\frac{2x+y}{2x} = -1 - \frac{y}{2x}$
$F_y = x + 2y + 2\mu y = 0 \qquad \mu = -\frac{x}{2y} - 1$
$F_\mu = x^2 + y^2 - 4 = 0$

$\frac{x}{2y} + \cancel{1} = \cancel{1} + \frac{y}{2x}$

$\cancel{2}x^2 = \cancel{2}y^2$
$x = \pm y$

$y^2 + y^2 = 4 \qquad F_{xx} = 2 + 2\mu$
$y^2 = 2 \qquad F_{xy} = 1 \qquad D = (2+2\mu)^2 - 1 =$
$y = \pm\sqrt{2} \qquad F_{yy} = 2 + 2\mu$

critical pts. $(\sqrt{2}, \sqrt{2}), (-\sqrt{2}, \sqrt{2}), (-\sqrt{2}, -\sqrt{2}), (\sqrt{2}, -\sqrt{2})$ ✓

$(\sqrt{2}, \sqrt{2}; -\frac{3}{2})$ $(\cancel{-\sqrt{2}, \sqrt{2}; -\frac{1}{2}})$ $(-\sqrt{2}, -\sqrt{2}; -\frac{3}{2})$ $(\cancel{\sqrt{2}, -\sqrt{2}; -\frac{1}{2}})$

$F_{xx} < 0$ for a max

$$F_{xx} = 2 + 2\mu \Big|_{(\sqrt{2}, \sqrt{2})} = 2 + 2\left(-\frac{3}{2}\right) = -1 < 0$$

$$F_{xx} = 2 + 2\mu \Big|_{(-\sqrt{2}, \sqrt{2})} = 2 + 2\left(-\frac{1}{2}\right) = 1 > 0$$

$$HF = \begin{bmatrix} f_{xx} & f_{xy} & f_{x\mu} \\ f_{yx} & f_{yy} & f_{y\mu} \\ f_{\mu x} & f_{\mu y} & f_{\mu\mu} \end{bmatrix} = \begin{bmatrix} 2+2\mu & 1 & 2x \\ 1 & 2+2\mu & 2y \\ 2x & 2y & 0 \end{bmatrix}$$

$d_2 = (2+2\mu)^2 - 1 = 0$ for $(\sqrt{2}, \sqrt{2})$

$f_{max} = f(\sqrt{2}, \sqrt{2}) = (2 + 2 + 2) = 6$

$\boxed{f_{max} = 6 \text{ at } (\sqrt{2}, \sqrt{2}) \text{ and } (-\sqrt{2}, -\sqrt{2})}$ ✓

21. Consider a cylindrical can. Use Lagrange mult. to determine the ratio between the dimensions of the can with the largest capacity.

$V = \pi r^2 h \qquad g(r,h) = 2\pi r^2 + 2\pi r h - c$

$F(r,h,\mu) = \pi r^2 h + \mu(2\pi r^2 + 2\pi r h - c)$

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$F_r = \cancel{2}\cancel{\pi} r h + \overset{2}{\cancel{4}} \mu \cancel{\pi} r + \cancel{2}\cancel{\pi} h \mu = 0$

$F_h = \cancel{\pi} r^{\cancel{2}} + 2\cancel{\pi}\cancel{r}\mu = 0$

$F_\mu = 2\pi r^2 + 2\pi r h - c = 0$

$\mu = -\frac{rh}{2r+h} \qquad \mu = -\frac{r}{2}$

$\frac{\cancel{r}h}{2r+h} = \frac{\cancel{r}}{2}$

$2h = 2r + h$

$h = 2r$

$\boxed{h = \text{diameter}}$ ✓

23. $M(x,y,z) = xz - y^2 + 3x + 3 \qquad g(x,y) = x^2 + y^2 + z^2 - 9$

$F(x,y,z,\mu) = xz - y^2 + 3x + 3 + \mu(x^2 + y^2 + z^2 - 9)$

$F_x = z + 3 + 2x\mu = 0 \qquad x + 2(-3-2x) = 0$

$F_y = -2y + 2y\mu = 0 \qquad \mu = 1 \qquad x - 6 - 4x = 0 \qquad z = -2(-2) - 3$

$F_z = x + 2z\mu = 0$ or $\qquad -3x = -6 \qquad z = 1$

$F_\mu = x^2 + y^2 + z^2 - 9 = 0 \quad y = 0 \Rightarrow$ more cases to check $\quad x = -2$

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$4 + y^2 + 1 = 9$

$y = \pm 2$

crit points: $(-2, 2, 1)$ $(-2, -2, 1)$

$F_{xx} = 2\mu = 2$

$F_{xy} = 0$

$F_{yy} = -2 + 2\mu = 0$

$F_{xz} = 1$

$F_{zz} = 2\mu = 2$

$F_{x\mu} = 2x = -3$

$F_{y\mu} = 2y = 3\sqrt{3}$

$F_{z\mu} = 2z = 0$

$F_{\mu\mu} = 0$

$F_{yz} = 0$

$$HF = \begin{bmatrix} 2 & 0 & 1 & -3 \\ 0 & 0 & 0 & 3\sqrt{3} \\ 1 & 0 & 2 & 0 \\ -3 & 3\sqrt{3} & 0 & 0 \end{bmatrix}$$

$d_1 = 2 > 0$

$d_2 = 0$

$d_3 = 0$

So the points $(-2, 2, 1)$ and $(-2, -2, 1)$ are the weakest field strength pts.

29. (a) find crit pts. of $f(x,y) = x+y$; $xy = 6$

$F(x,y,\mu) = x+y + \mu(xy-6)$

$F_x = 1 + \mu y = 0 \qquad \mu = -\frac{1}{y} \qquad x = y$

$F_y = 1 + \mu x = 0 \qquad \mu = -\frac{1}{x}$

$F_\mu = xy - 6 = 0$

$x^2 - 6 = 0$

$x^2 = 6$

$x = \pm\sqrt{6}$

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crit. points @ $(\sqrt{6}, \sqrt{6})$, $(-\sqrt{6}, -\sqrt{6})$ ✓

(b) The graph $xy = 6$ is a hyperbola, so it is not bounded. It goes on infinitely in the x and y direction. ✓

$n = 8$

6(a)

x	y	xy	x^2
8	85	680	64
8.5	72	612	72.25
9	95	855	81
7	68	476	49
4	52	208	16
8.5	75	637.5	72.25
7.5	90	675	56.25
6	65	390	36
$\Sigma x = 58.5$	$\Sigma y = 602$	4533.5	446.75

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$$m = \frac{n\,\Sigma xy - (\Sigma x)(\Sigma y)}{n\,\Sigma x^2 - (\Sigma x)^2} = \frac{8(4533.5) - (58.5)(602)}{8(446.75) - (58.5)^2}$$

$m = 6.92586$ ✓

$$b = \frac{(\Sigma x^2)(\Sigma y) - (\Sigma x)(\Sigma xy)}{n\,\Sigma x^2 - (\Sigma x)^2} = \frac{(446.75)(602) - (58.5)(4533.5)}{8(446.75) - (58.5)^2}$$

$b = 24.60461$ ✓

(c)

$y = 6.92586x + 24.60461$

$y = 6.92586(6.8) + 24.60461$

$y = 71.7$

$\boxed{y \approx 72}$ ✓