Fang-Yuan Chang
Math 61 Sec 2
HW # 5
Due: 5/25/2004

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Tot Pg 9

Section 5.5 5, 7, 24, 29

Section 6.2 4, 13, 15, 17, 20

Sec 5.5

5) If $T(u,v,w) = (3u - v, u - v + 2w, 5u + 3v - w)$ describe how T transforms the unit cube $W^* = [0,1] \times [0,1] \times [0,1]$

We can rewrite T by using matrix multiplication

$\frac{10}{10}$

$$T(u,v,w) = \begin{bmatrix} 3 & -1 & 0 \\ 1 & -1 & 2 \\ 5 & 3 & -1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Note that if

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 1 & -1 & 2 \\ 5 & 3 & -1 \end{bmatrix}$$

then $\det A \overset{TI\text{-}83}{=} -26 \neq 0$

so T is one-one and onto, and T maps parallelopiped to parallelpipeds.
In particular, the unit cube

$W^* = [0,1] \times [0,1] \times [0,1]$

is mapped onto some parallelopiped $W = T(W^*)$ with volume

$|\det A| \cdot \text{volume of } D^* = |-26| \cdot 1 = 26$

The images of the vertices of the cube are

$$T(0,0,0) = \begin{bmatrix} 3 & -1 & 0 \\ 1 & -1 & 2 \\ 5 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} + 0 \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \underline{(0,0,0)} \checkmark$$

$T(0,0,1) = \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} = (0,2,-1)$ ✓

$T(1,1,0) = (2,0,8)$ ✓

$T(1,1,1) = (2,2,7)$ ✓

$T(0,1,1) = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \underline{(-1,1,2)}$ ✓

$T(0,1,0) = (-1,-1,3)$ ✓

$T(1,0,1) = \underline{(3,3,4)}$ ✓

$T(1,0,0) = \underline{(3,1,5)}$ ✓

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To summarize,

$T(0,0,0) \to (0,0,0)$ $\quad T \to (1,0,0) \to (3,1,5)$

$T(0,1,0) \to (-1,-1,3)$ $\quad T \to (0,0,1) \to (0,2,-1)$

$T\ (1,1,0) \to (2,0,8)$ $\quad T(1,0,1) \to (3,3,4)$

$T(0,1,1) \to (-1,1,2)$ $\quad T(1,1,1) \to (2,2,7)$

and these vertices uniquely describe the parallelepiped D.

7) Let $T\ \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by

$$T(\rho, \varphi, \theta) = (\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi)$$

$\frac{10}{10}$

a) Determine $D = T(D^*)$, where $D^* = [0,1] \times [0,\pi] \times [0, 2\pi]$

$$(x,y,z) = (\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi)$$

Notice that they correspond to spherical coordinates.

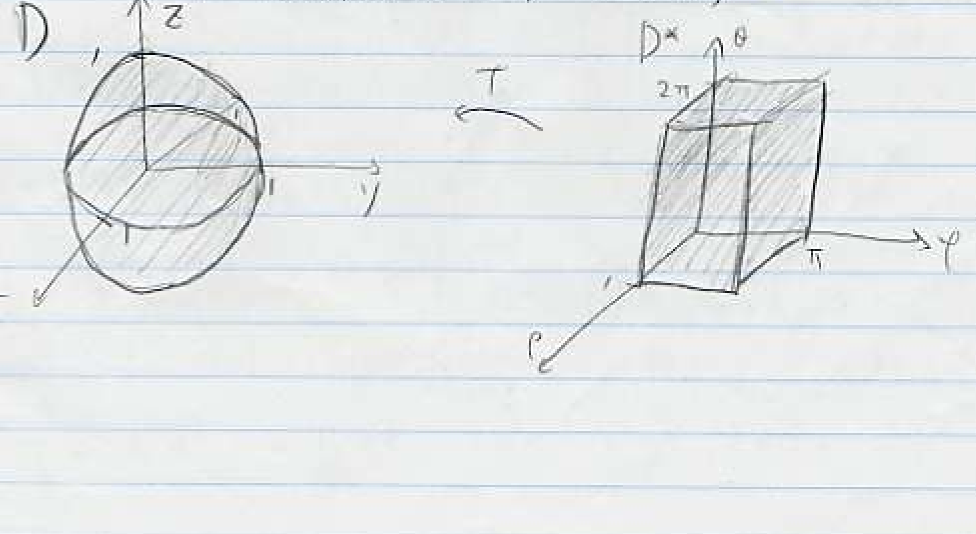
$\rho : [0,1]$ $\qquad \varphi : [0,\pi]$ $\qquad \theta : [0, 2\pi]$

$0 \le \rho \le 1$, $\quad 0 \le \varphi \le \pi$, $\quad 0 \le \theta \le 2\pi$,

which are ranges for a uniform sphere with a radius of 1.

D can thus be described as

$$D = \{ (x,y,z) \mid x^2 + y^2 + z^2 \le 1 \}$$ ✓



b) Determine $D = T(D^*)$ when

$$D^* = [0,1] \times [0, \pi/2] \times [0, \pi/2]$$

$$(x,y,z) = (\rho \sin\varphi \cos\theta,\ \rho \sin\varphi \sin\theta,\ \rho\cos\varphi)$$

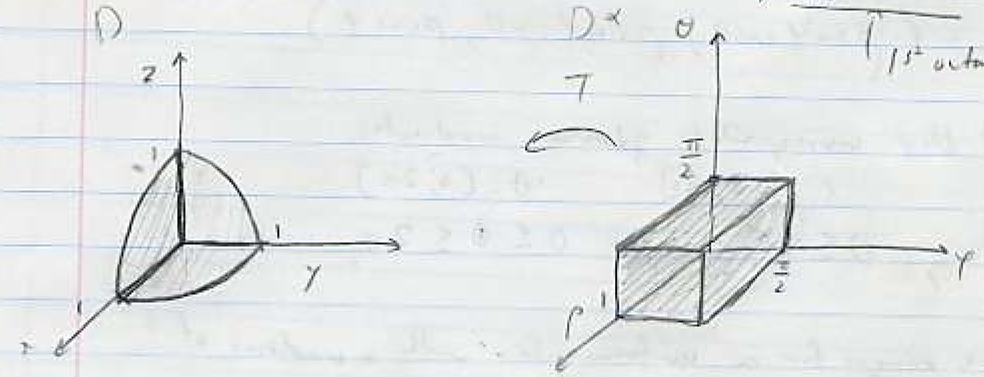
$\rho : [0,1]$ $\qquad$ $\varphi : [0, \pi/2]$ $\qquad$ $\theta : [0, \pi/2]$

$0 \le \rho \le 1$ $\qquad$ $0 \le \varphi \le \pi/2$ $\qquad$ $0 \le \theta \le \pi/2$

Notice spherical coordinates, except now the ranges φ and θ are such that only the 1st octant portion of the unit sphere remains (half the φ, quarter the θ). The image D is then a quarter sphere of radius 1.

D can then be described as

$D = \{ (x,y,z) \mid x^2+y^2+z^2 \le 1,\ \underbrace{x, y, z \ge 0}_{\text{1st octant}} \}$ ✓



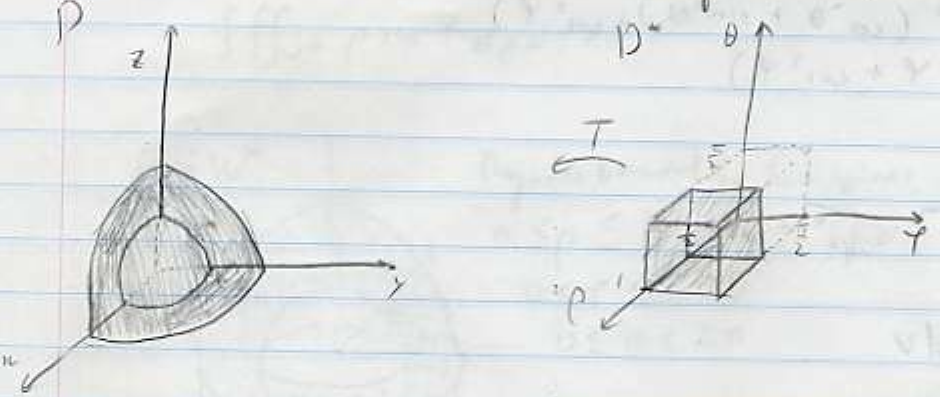
2c) Determine $D = T(D^*)$, where $D^* = [\frac{1}{2}, 1] \times [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$

$(x,y,z) = (\rho \sin\varphi \cos\theta,\ \rho\sin\varphi\sin\theta,\ \rho\cos\varphi)$

$\rho : [\frac{1}{2}, 1]$ $\qquad \varphi\ [0, \frac{\pi}{2}]$ $\qquad \theta : [0, \frac{\pi}{2}]$

$\frac{1}{2} \le \rho \le 1$ $\qquad 0 \le \varphi \le \pi/2$ $\qquad 0 \le \theta \le \pi/2$

Notice that because the φ and θ ranges are the same as that of part b the image D will look like a quarter sphere
However, the radius of the sphere, determined by the range of ρ, only spans from $\frac{1}{2}$ to 1 instead of 0 to 1, making the sphere hollow from 0 to $\frac{1}{2}$.
Thus, the image of D is a quarter sphere of radius 1 with a hole of a size of a quarter sphere of radius $\frac{1}{2}$



D can be described as

$D = \{(x,y,z) \mid \underbrace{\frac{1}{4} \le x^2+y^2+z^2 \le 1}_{\text{radius } \frac{1}{2} \text{ to } 1},\ \underbrace{x, y, z \ge 0}_{1\text{st octant}}\}$ ✓

15/15

24) determine the values of the given integrals where W is the region bounded by the two spheres $x^2+y^2+z^2=a^2$ and $x^2+y^2+z^2=b^2$, for $0<a<b$

Use spherical coordinates

Let

$$\begin{cases} x = \rho \sin\varphi \cos\theta \\ y = \rho \sin\varphi \sin\theta \\ z = \rho\cos\varphi \end{cases}$$

$$x^2+y^2+z^2 = \rho^2 \sin\varphi \cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta + \rho^2\cos^2\varphi$$

$$= \rho^2(\sin^2\varphi\cos^2\theta + \sin^2\varphi\sin^2\theta + \cos^2\varphi)$$
$$= \rho^2(\sin^2\varphi(\cos^2\theta+\sin^2\theta)+\cos^2\varphi)$$
$$= \rho^2(\sin^2\varphi+\cos^2\varphi)$$
$$= \underline{\rho^2}$$

$$\sqrt{x^2+y^2+z^2} = \sqrt{\rho^2} = \rho$$

<u>Determine</u> dV

$dV = dx\,dy\,dz$

has Jacobian

$$\frac{\partial(x,y,z)}{\partial(\rho,\varphi,\theta)} = \det\begin{bmatrix} x_\rho & x_\varphi & x_\theta \\ y_\rho & y_\varphi & y_\theta \\ z_\rho & z_\varphi & z_\theta \end{bmatrix} = \det\begin{bmatrix} \sin\varphi\cos\theta & \rho\cos\varphi\cos\theta & -\rho\sin\varphi\sin\theta \\ \sin\varphi\sin\theta & \rho\cos\varphi\sin\theta & \rho\sin\varphi\cos\theta \\ \cos\varphi & -\rho\sin\varphi & 0 \end{bmatrix}$$

Using cofactor expansion about the last row, the determinant is equal to

$$\cos\varphi(\rho^2\cos^2\theta\sin\varphi\cos\varphi + \rho^2\sin^2\theta\sin\varphi\cos\varphi) + \rho\sin\varphi(\rho\cos^2\theta\sin^2\varphi + \rho\sin^2\theta\sin^2$$
$$= \rho^2\cos\varphi(\sin\varphi\cos\varphi) + \rho^2\sin^2\varphi$$
$$= \rho^2\sin\varphi(\cos^2\varphi+\sin^2\varphi)$$
$$= \rho^2\sin\varphi \checkmark$$

FT 4

We want to find $\left| \frac{\partial(x,y,z)}{\partial(\rho,\varphi,\theta)} \right|$, and we can see that under the restriction of the spherical coordinates of $0 \le \varphi \le \pi$, $\sin\varphi$ will always be nonnegative. Thus, the Jacobian will also be nonnegative.

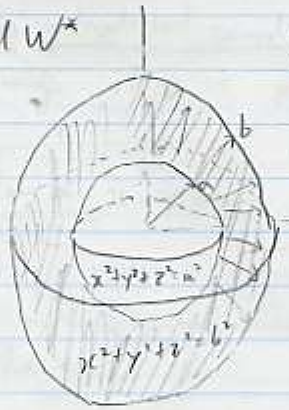
Therefore, the volume element in spherical coordinates is
$dV = \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$.

And by the change of variables formula

$$\iiint_W \frac{dV}{\sqrt{x^2+y^2+z^2}} = \iiint_{W^*} \frac{\rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta}{\rho} =$$

$$\iiint_{W^*} \rho \sin\varphi \, d\rho \, d\varphi \, d\theta$$

Find W^*



Region is bounded by two spheres, one with radius a and another with b, so

$a \le \rho \le b$ Since $0 < a < b$

$0 \le \varphi \le \pi$

$0 \le \theta \le 2\pi$

$$\iiint_{W^*} \rho \sin\varphi \, d\rho \, d\varphi \, d\theta = \int_0^{2\pi}\int_0^{\pi}\int_a^b \rho \sin\varphi \, d\rho \, d\varphi \, d\theta =$$

$$\iint \frac{\rho^2 \sin\varphi}{2}\Big|_a^b d\varphi \, d\theta = \int_0^{2\pi}\int_0^{\pi} \frac{b^2}{2}\sin\varphi - \frac{a^2}{2}\sin\varphi \, d\varphi \, d\theta$$

$$= \frac{b^2}{2}\int_0^{2\pi}\int_0^{\pi} \sin\varphi \, d\varphi \, d\theta - \frac{a^2}{2}\int_0^{2\pi}\int_0^{\pi} \sin\varphi \, d\varphi \, d\theta = \frac{b^2}{2}\int_0^{2\pi} -\cos\varphi\Big|_0^{\pi} d\theta - \frac{a^2}{2}\int_0^{2\pi} -\cos\varphi\Big|_0^{\pi} d\theta$$

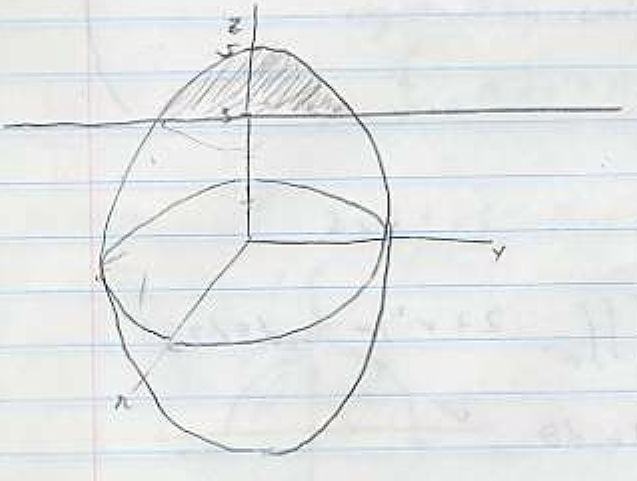
$$= \frac{b^2}{2}\int_0^{2\pi} -(-1-1)\,d\theta - \frac{a^2}{2}\int_0^{2\pi} -(-1-1)\,d\theta = \frac{b^2}{2}\int_0^{2\pi} 2\,d\theta - \frac{a^2}{2}\int_0^{2\pi} 2\,d\theta$$

$$= b^2\int_0^{2\pi} d\theta - a^2\int_0^{2\pi} d\theta = 2b^2\pi - 2a^2\pi = \boxed{2\pi(b^2-a^2)}$$ ✓

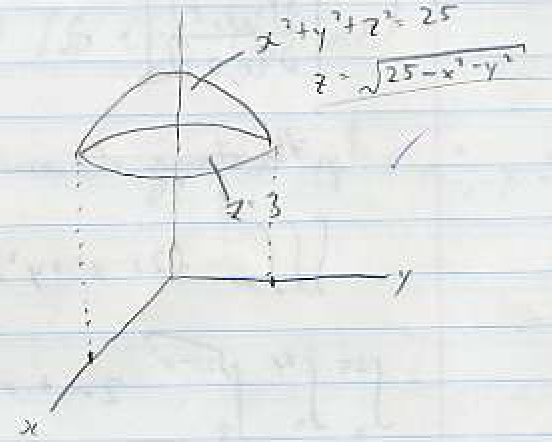
FY. 5

29) Find
$$\iiint_W (2+x^2+y^2)\, dV,$$

where W is the region inside the sphere $x^2+y^2+z^2=25$ and above the plane $z=3$

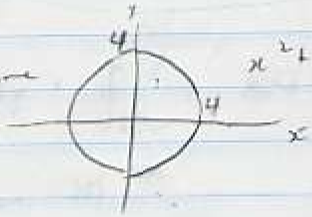


$\longrightarrow$



Using cylindrical coordinates,
the range of z is $z=3 \longrightarrow z=\sqrt{25-x^2-y^2}=\sqrt{25-(r^2\cos^2\theta+r^2\sin^2\theta)}=\sqrt{25-r^2}$

Shadow in x-y plane



$x^2+y^2+z^2=25 \quad z=3, \; x^2+y^2+3^2=25$

$x^2+y^2=16$ radius of 4

$r: \; 0\to 4 \quad \theta: 0 \to 2\pi$

where
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$2+x^2+y^2 = 2+r^2\cos^2\theta+r^2\sin^2\theta = 2+r^2(\cos^2\theta+\sin^2\theta) = 2+r^2$

Determine dV. $dV = dx\,dy\,dz$
has Jacobian

$$\frac{\partial(x,y,z)}{\partial(r,\theta,z)} = \det\begin{bmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = r\cos^2\theta + r\sin^2\theta = r.$$

so we see that the volume element in cylindrical coordinates is

$dV = r\,dr\,d\theta\,dz$

(Recall that the cylindrical coordinate r is usually taken to be nonnegative by convention. In our case, $r: 0 \to 5$, which is nonnegative so

$$\left|\frac{\partial(x,y,z)}{\partial(r,\theta,z)}\right| = r.$$)

$\frac{15}{15}$

By the change of variables formula

$$\iiint_W (2+x^2+y^2)\,dV = \iiint_{W^*} (2+r^2)r\,dr\,d\theta\,dz$$

$$= \int_0^{2\pi}\int_0^4\int_3^{\sqrt{25-r^2}} 2r + r^3 \; dz\,dr\,d\theta$$ ✓

$\overset{TI\text{-}89}{=} \frac{656\pi}{5}$ ✓

F.Y.
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Sec 6.2.

4) Verify Green's theorem for the given vector field

$\vec{F} = M(x,y)\,i + N(x,y)\,j$ and

region D by calculating both

$\oint_{\partial D} M\,dx + N\,dy$ and $\iint_D (N_x - M_y)\,dA$

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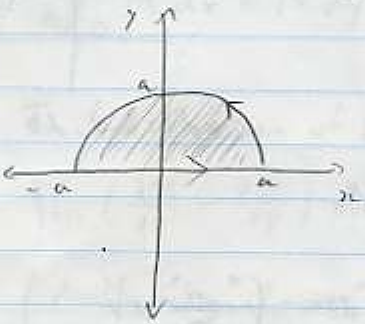
$\vec{F} = 2y\,i + x\,j$, D is the semicircular region $x^2+y^2 \le a^2$, $y \ge 0$.



Calculate

$$\oint_{\partial D} M\,dx + N\,dy = \oint_{\partial D} 2y\,dx + x\,dy$$

Parametrize the curve ∂D

∂D $\begin{cases} x = a\cos t \\ y = a\sin t \end{cases}$ $\quad 0 \le t \le \pi \quad$ and $\begin{cases} x = t \\ y = 0 \end{cases}$ $\quad t: -a \to a$

$$\oint_{\partial D} 2y\,dx + x\,dy = \int_0^\pi 2a\sin t\;d(a\cos t) + \int_0^\pi a\cos t\;d(a\sin t)$$
$$+ \int_{-a}^{a} 0\,dt + t\,d0$$ → which is zero

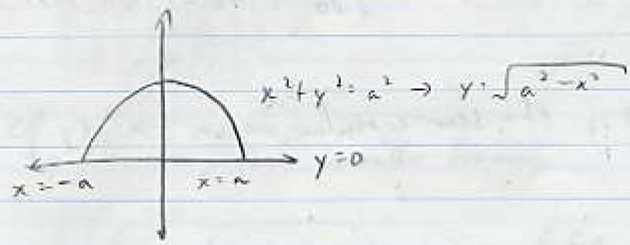
$d(a\cos t) = -a\sin t\,dt \qquad d(a\sin t) = a\cos t\,dt$

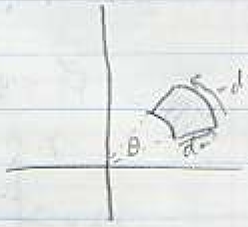
$$\int_0^\pi 2a\sin t\,(-a\sin t)\,dt + \int_0^\pi a\cos t\,(a\cos t)\,dt$$
$$= -2a^2\int_0^\pi \sin^2 t\,dt + a^2\int_0^\pi \cos^2 t\,dt$$
$$= -2a^2\int_0^\pi \frac{1-\cos 2t}{2}\,dt + a^2\int_0^\pi \frac{1+\cos 2t}{2}\,dt$$
$$= -2a^2\left(\frac{t}{2} - \frac{\sin(2t)}{4}\right)\Big|_0^\pi + a^2\left(\frac{t}{2} + \frac{\sin(2t)}{4}\right)\Big|_0^\pi = -2a^2\left(\frac{\pi}{2}\right) + a^2\left(\frac{\pi}{2}\right) = \boxed{-\frac{a^2\pi}{2}}$$

$$\iint_D (N_x - M_y)\,dA = \iint_D (1-2)\,dA = \iint_D -1\,dA = -\iint_D dA$$

$N = x \quad N_x = 1$

$M = 2y \quad M_y = 2$





$dx\,dy = r\,dr\,d\theta$

$$-\iint_D dA = -\int_{-a}^{a}\int_0^{\sqrt{a^2-x^2}} dy\,dx$$

Change of variables

$$= -\int_0^{\pi}\int_0^{a} r\,dr\,d\theta = -\int_0^{\pi} \frac{r^2}{2}\Big|_0^a d\theta = -\int_0^{\pi} \frac{a^2}{2}\,d\theta$$

$$= \boxed{-\frac{a^2\pi}{2}}$$

F.Y.
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13) Show that if D is a region to which Green's theorem applies and ∂D is oriented so that D is always on the left as we travel along ∂D, then the area of D is given by either of the following two line integrals

Area of $D = \oint_{\partial D} x\,dy = -\oint_{\partial D} y\,dx$

By Green's theorem, if the above conditions apply, then

$$\oint_{\partial D} M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

To find the area of D, we can find

$\iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$, where the function $\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 1$

(double integral over the function 1 calculates the area)

Let $\frac{\partial N}{\partial x} = 1$ and $\frac{\partial M}{\partial y} = 0$, which satisfies the equation

$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 1 - 0 = 1$

$\frac{10}{10}$

Then

N can equal to $\underline{x}$ and M can equal to $\underline{0}$

(Notice the "can equal", which means that there are many other possibilities of N and M).

So by substituting into the left hand side of the Green's theorem

$\oint_{\partial D} M\,dx + N\,dy = \oint_{\partial D} 0\,dx + x\,dy = \underline{\int_{\partial D} x\,dy}$

So the area of D can be found by $\oint_{\partial D} x\,dy$ ✓

We can also let $\frac{\partial N}{\partial x} = 0$ and $\frac{\partial M}{\partial y} = -1$, which also satisfies the equation

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 0 - (-1) = 1$$

Then,

N can equal to 0 and M can equal to $-y$

So by substituting into the left hand side of the Green's theorem,

$$\oint_{\partial D} M\,dx + N\,dy = \oint_{\partial D} -y\,dx + 0\,dy = \oint_{\partial D} -y\,dx = -\oint_{\partial D} y\,dx$$

so the area of D can also be found by $-\oint_{\partial D} y\,dx$

Thus

area of D $= \oint_{\partial D} x\,dy = -\oint_{\partial D} y\,dx$ ∎

K.T.

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15) a) Use the divergence theorem to show that

$\oint_c \vec{F} \cdot \hat{n}\, ds = 0$, where

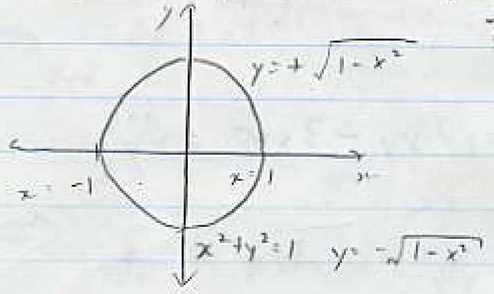
$\vec{F} = 2y\hat{\imath} - 3x\hat{\jmath}$ and C is the circle $x^2+y^2=1$

By Divergence theorem,

$$\oint_C \vec{F}\cdot\hat{n}\, ds = \iint_D \nabla\cdot\vec{F}\, dA$$

Since C is the circle $x^2+y^2=1$

10/10



The limit of the region D is

$x: -1 \to 1$

$y: -\sqrt{1-x^2} \to \sqrt{1-x^2}$

$dA = dx\,dy$

Converting to polar coordinates gives

$dx\,dy = r\,dr\,d\theta$

$r: 0 \to 1$

$\theta: 0 \to 2\pi$

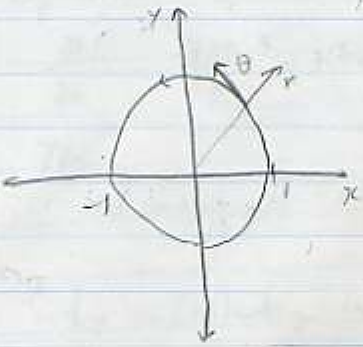
So $$\oint_C \vec{F}\cdot\hat{n} = \iint_D \nabla\cdot\vec{F}\, dA = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \nabla\cdot\vec{F}\, dy\, dx$$

$$= \int_0^{2\pi}\int_0^1 (\nabla\cdot\vec{F})\, r\, dr\, d\theta$$

$$= \nabla\cdot\vec{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\cdot(2y, -3x) = \frac{\partial 2y}{\partial x} + \frac{\partial -3x}{\partial y} = 0 + 0 = \boxed{0}$$

$$\Rightarrow \int_0^{2\pi}\int_0^1 (\nabla\cdot\vec{F})\, r\, dr\, d\theta = \int_0^{2\pi}\int_0^1 0\cdot r\, dr\, d\theta = \boxed{0}$$

15 b) Now show $\oint_C \vec{F}\cdot\hat{n}\, ds = 0$ by direct computation of the line integral



$\hat{n} = \hat{r} = \frac{x\hat{\imath} + y\hat{\jmath}}{\text{magnitude}} = \frac{x\hat{\imath}+y\hat{\jmath}}{1}$

$\hat{n} = x\hat{\imath} + y\hat{\jmath}$
$= (x, y)$

$\vec{F} = 2y\hat{\imath} - 3x\hat{\jmath} = (2y, -3x)$

$\vec{F}\cdot\hat{n} = (2y, -3x)\cdot(x, y) = 2xy - 3xy = -xy$

Parametrize the curve

$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad 0 \le t \le 2\pi$

$ds = \| \vec{x}'(t) \| \, dt$

$\vec{x}(t) = (\cos t, \sin t)$

$x'(t) = (-\sin t, \cos t)$

$\|x'(t)\| = \sqrt{\sin^2 t + \cos^2 t} = 1$

$ds = 1 \cdot dt$

$\oint_C \vec{F}\cdot\hat{n}\, ds = \oint_C -xy\, ds = \int_0^{2\pi} -\cos t \sin t\, dt$

$= -\int_0^{2\pi} \sin t \cos t\, dt = -\int_0^{2\pi} \frac{\sin 2t}{2}\, dt = -\left(-\frac{\cos 2t}{4}\right)_0^{2\pi}$

$= -\left(-\frac{\cos 4\pi}{4} + \frac{\cos 0}{4}\right) = -(-1+1) = \boxed{0}$

F.Y.
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17) Let C be any simple, closed curve in the plane. Show that

$$\oint_C 3x^2 y\, dx + x^3 dy = 0.$$

By Green's theorem

$$\oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

where D is the region bounded by the curve C

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for the equation above

$M = 3x^2 y$

$N = x^3$

and

$$\frac{\partial N}{\partial x} = 3x^2 \qquad \frac{\partial M}{\partial y} = 3x^2$$

$$\Rightarrow \oint_C 3x^2 y\, dx + x^3 dy =$$

$$\iint_D (3x^2 - 3x^2)\, dx\,dy = \iint_D 0\, dx\,dy = \underline{\underline{0}}.$$ ✓

20) Let $f(x, y)$ be a function of class C^2 such that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$$

10/10

Show that if C is any closed curve to which Green's theorem applies, then

$$\oint_C \frac{\partial f}{\partial y} dx - \frac{\partial f}{\partial x} dy = 0$$

By Green's theorem

$$\oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

where D is the region bounded by the curve C

for the equation above

$M = \frac{\partial f}{\partial y}$ and $N = -\frac{\partial f}{\partial x}$

so $\frac{\partial M}{\partial y} = \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial N}{\partial x} = -\frac{\partial^2 f}{\partial x^2}$

$\Rightarrow$ by Green's theorem

$$\oint_c \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = \iint_D \left(-\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right) dx\,dy$$

$$= \iint_D -\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) dx\,dy$$

but it is given that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

so by substitution

$$\oint_c \frac{\partial f}{\partial y}\,dx - \frac{\partial f}{\partial x}\,dy = \iint_D -0\,dx\,dy = 0 \quad \checkmark$$