p 418: 9, 11, 13, 15, 21

$\frac{100}{100}$!

M. Jameson
Math 61 Section 1
HW # 8
Due 5/28/04

(9) Let S be closed cylinder def. by $z=0$, $z=4$, and $x^2+y^2=9$, with outward normal

$$\iint_S y\,dS = \iint_{top} y\,dS + \iint_{bottom} y\,dS + \iint_{lat.\ surface} y\,dS$$

<u>Top</u> Par. $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 4 \end{cases}$ $\begin{matrix} 0 \le r \le 3 \\ 0 \le \theta \le 2\pi \end{matrix}$ $\|T_r \times T_\theta\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} \right\| = \|(0,0,r)\| = r$

$$\iint_{top} y\,dS = \int_0^{2\pi}\int_0^3 r^2\sin\theta\,dr\,d\theta = \frac{3^3}{3}\int_0^{2\pi}\sin\theta\,d\theta = -9\cos\theta\Big|_0^{2\pi} = 0$$

$\frac{15}{15}$

<u>Bottom</u> Par. $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 0 \end{cases}$ $\begin{matrix} 0 \le r \le 3 \\ 0 \le \theta \le 2\pi \end{matrix}$ $\|T_r \times T_\theta\| = r$ as in last part. (although vector points opposite)

$$\iint_{bottom} y\,dS = \int_0^{2\pi}\int_0^3 r^2\sin\theta\,dr\,d\theta = 0$$

<u>Lat. Surf.</u> Par $\begin{cases} x = 3\cos\theta \\ y = 3\sin\theta \\ z = h \end{cases}$ $\begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le h \le 4 \end{matrix}$ $\|T_\theta \times T_h\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \right\| = \|(3\cos\theta, 3\sin\theta, 0)\| = 3$

$$\int_0^{2\pi}\int_0^4 3\sin\theta\,(3\,dh\,d\theta) = 36\int_0^{2\pi}\sin\theta\,d\theta = 36\left(-\cos\theta\Big|_0^{2\pi}\right) = 0$$

$$\iint_S y\,dS = \iint_{top} y\,dS + \iint_{bottom} y\,dS + \iint_{L.S.} y\,dS = 0 + 0 + 0 = \boxed{0}$$

(11) For same S, $\iint x^2\,dS$

<u>Top</u>: Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 4 \end{cases}$ $\begin{matrix} 0 \le r \le 3 \\ 0 \le \theta \le 2\pi \end{matrix}$ $\|T_r \times T_\theta\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ -\sin\theta & r\cos\theta & 0 \end{vmatrix} \right\| = r$

$\frac{20}{20}$

$$\iint_{top} x^2\,dS = \int_0^{2\pi}\int_0^3 r^2\cos^2\theta\,r\,dr\,d\theta = \frac{3^4}{4}\int_0^{2\pi}\cos^2\theta\,d\theta = \frac{81}{4}\pi$$

<u>Bottom</u> Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 0 \end{cases}$ $\begin{matrix} 0 \le r \le 3 \\ 0 \le \theta \le 2\pi \end{matrix}$ $\|T_r \times T_\theta\| = r$

$$\iint_{bot} x^2\,dS = \int_0^{2\pi}\int_0^3 r^2\cos^2\theta\,r\,dr\,d\theta = \frac{81}{4}\pi$$

<u>Lat Surf.</u> Par. $\begin{cases} x = 3\cos\theta \\ y = 3\sin\theta \\ z = h \end{cases}$ $\begin{matrix} 0 \le h \le 4 \\ 0 \le \theta \le 2\pi \end{matrix}$ $\|T_h \times T_\theta\| = \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ -3\sin\theta & 3\cos\theta & 0 \end{vmatrix} \right\| = 3$

(11) Con't $\iint_{LS} x^2 dS = \int_0^4\int_0^{2\pi} 27\cos^2\theta \, d\theta \, dh = 27 \cdot 4 \int_0^{2\pi} \cos^2\theta \, d\theta = 108\pi$

$$\iint_S x^2 dS = \frac{81}{4}\pi + \frac{81}{4}\pi + 108\pi = \boxed{\frac{297}{2}\pi}$$

(13) For Same S, $\iint_S z\vec{k} \cdot \vec{dS}$ ← vector

Top Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 4 \end{cases}$ $T_r \times T_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (0, 0, r)$ ← good, should be pointing out (up).

$$\iint_{top} z\vec{k} \cdot \vec{dS} = \int_0^{2\pi}\int_0^3 zr \, dr \, d\theta = 4 \cdot \frac{3^2}{2}\int_0^{2\pi} d\theta = 36\pi$$

Bottom Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 0 \end{cases}$ we know $\iint_{bottom} z\vec{k} \cdot \vec{dS} = \iint 0 \, \vec{k} \cdot \vec{dS} = 0$

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LS Par $\begin{cases} x = 3\cos\theta & 0 \le \theta \le 2\pi \\ y = 3\sin\theta & 0 \le h \le 4 \\ z = h \end{cases}$ $T_\theta \times T_h = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (3\cos\theta, 3\sin\theta, 0)$

$$\iint_{LS} z\vec{k} \cdot \vec{dS} = \iint z(0,0,1) \cdot (3\cos\theta, 3\sin\theta, 0) \, dS = 0$$

$$\iint_S z\vec{k} \cdot \vec{dS} = 36\pi + 0 + 0 = \boxed{36\pi}$$

(15) For Same S, $\iint_S (-y, x, 0) \cdot \vec{dS}$

Top Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 4 \end{cases}$ $T_r \times T_\theta = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & 0 \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = (0, 0, r)$

$$\iint_{top} (-y, x, 0) \cdot \vec{dS} = \int_0^{2\pi}\int_0^3 (-y, x, 0) \cdot (0, 0, r) \, dr \, d\theta = 0$$

Bottom Par $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 0 \end{cases}$ $T_r \times T_\theta = (0, 0, r)$, so $\iint_{bottom} (-y, x, 0) \vec{dS} = 0$ as for top

15/15

LS Par $\begin{cases} x = 3\cos\theta \\ y = 3\sin\theta \\ z = h \end{cases}$ $T_\theta \times T_h = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (3\cos\theta, 3\sin\theta)$

$$\iint_{LS} (-y, x, 0) \cdot \vec{dS} = \int_0^4\int_0^{2\pi} (-3\sin\theta, 3\cos\theta, 0) \cdot (3\cos\theta, 3\sin\theta, 0) \, d\theta \, dh = 0$$

Thus: $\iint_S (-y, x, 0) \cdot \vec{dS} = 0 + 0 + 0 = \boxed{0}$

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HW#8
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(21) Let S be a funnel-shaped surface given by $x^2+y^2=z^2$ for $1\le z\le 9$ and $x^2+y^2=1$ for $0\le z\le 1$

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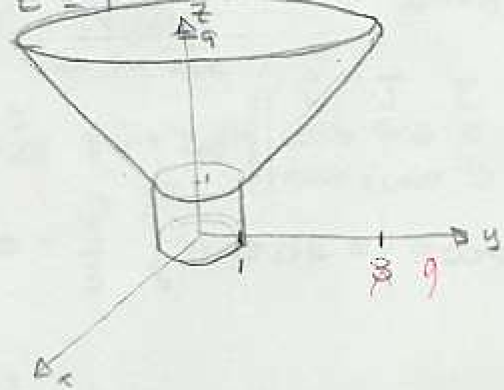
(A) Sketch S



(B) Determine outward pointing unit normal vectors to S

For $0\le z\le 1$: $\begin{cases} x=\cos\theta & 0\le\theta\le 2\pi \\ y=\sin\theta & 0\le h\le 1 \\ z=h \end{cases}$

$$T_\theta \times T_h = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos\theta, \sin\theta, 0) = \boxed{(x,y,0)}$$ For $0\le z\le 1$ ✓

For $1\le z\le 9$ $\begin{cases} x=h\cos\theta & 0\le\theta\le 2\pi \\ y=h\sin\theta & 1\le h\le 9 \\ z=h \end{cases}$

$$T_\theta \times T_h = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -h\sin\theta & h\cos\theta & 0 \\ \cos\theta & \sin\theta & 1 \end{vmatrix} = (h\cos\theta, h\sin\theta, -h\sin^2\theta - h\cos^2\theta)$$

$= (x, y, -z)$ ✓ ← now we need to normalize

$$\frac{(x,y,-z)}{\|(x,y,-z)\|} = \frac{(x,y,-z)}{\sqrt{x^2+y^2+z^2}} = \frac{(x,y,-z)}{\sqrt{2z^2}} = \boxed{\left(\frac{x}{z\sqrt{2}}, \frac{y}{z\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)}$$ For $1\le z\le 9$ ✓

(C) Evaluate $\iint_S \vec{F}\cdot d\vec{S}$ where $\vec{F}=(-y,x,z)$ w/ outward normals

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_{\text{bottom}} \vec{F}\cdot d\vec{S} + \iint_{\text{top}} \vec{F}\cdot d\vec{S} = \int_0^1\int_0^{2\pi} (-y,x,z)\cdot(x,y,0)\,d\theta\,dh + \int_1^9\int_0^{2\pi} (-y,x,z)\cdot(x,y,-z)\,d\theta\,dh$$

$$= \int_0^1\int_0^{2\pi} 0\,d\theta\,dh + \int_1^9\int_0^{2\pi} -z^2\,d\theta\,dh = -2\pi\left(\frac{z^3}{3}\Big|_1^9\right) = \boxed{\frac{-1456}{3}\pi}$$