MATH 12: HANDOUT DS1
(BUT WRITE THE HW NUMBER FROM THE WEBPAGE ON YOUR ASSIGNMENT)

0. Familiarize yourself with the *Discrete Tool* on *ODE Architect*. This is software which will help you analyze discrete dynamical systems. Go to the Kato labs, and login on the PC's. Then:

Click on *Application Launcher*.

Click on *Discrete Tool*.

In the Discrete Tool, remember that multiplication in equations must be denoted by a $*$.

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1. Use *graphical analysis* to analyze the dynamics of the linear recurrence

$$x_{n+1} = ax_n + b$$

in each of the following cases. Determine the long-term behavior of the system for all possible initial states x_0. (Are there fixed points? Periodic points? If so, find them. Do some orbits fly off to ∞ or $-\infty$? Do some orbits approach fixed points?)

When completed, check your answers using *Discrete Tool*.

(a) $a = 1, b > 0$.

(b) $a = -1$ and b any real number.

2. Use the *Discrete Tool* to answer the following questions. Let x_n = proportion of sick people each month n. The *logistic map* is a model for the spread of infection:

$$x_{n+1} = cx_n(1 - x_n).$$

The parameter c may be thought of as the transmission rate. (In Discrete Tool, this map is loaded up by default... just change the value of c to what you want.)

(a) Let $c = 2.9$. Choose any initial seed x_0 between 0 and 1.

Give BRIEF answers for the following questions.

Can you determine the long-term behavior of its iterates, and if so, what happens?

Give numerical estimates for any limiting values you detect.

Try several different seeds... does the long-term behavior change?

What do your results mean in terms of the spread of infection?

(b) Now let $c = 3.832$. Answer the same questions as in (a) above.

(c) Now let $c = 3.8$. Answer the same questions as in (a) above.

3. In this problem, you will prove the Mean Value Theorem in a series of guided steps. Let $a \neq b$. The Mean Value Theorem says: if $g : [a, b] \to \mathbf{R}$ is differentiable, then there exists a $c \in (a, b)$ such that

$$g'(c) = \frac{g(b) - g(a)}{b - a}. \tag{1}$$

To prove this, assume g is differentiable as given, and perform the following steps:

(a) What is the equation of the secant line through $(a, g(a))$ and $(b, g(b))$?

(b) Construct a new function f from g by setting

$$f(x) = g(x) - \left[g(a) + \frac{g(b) - g(a)}{b - a}(x - a)\right].$$

(Notice where the variable x appears. Everything else are just constants!) What does the bracketed term have to do with the secant line from part (a)?

(c) Evaluate $f(a)$, $f(b)$, and $f'(x)$.

(d) Can Rolle's theorem be applied to f? Now use this information to derive Equation (**??**). This proves the Mean Value Theorem.

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Problems **4** through **6** are on the Mean Value Theorem. They come from the text *Calculus: One Variable*, by Salas, Hille and Etgen.

4. Given $f(x) = x^3$ on the interval $[1, 3]$, verify that f satisfies the conditions of the mean-value theorem on the indicated interval, and find all numbers c that satisfy the conclusion of the theorem.

5. Set $f(x) = x^{-1}$ on the interval $[a, b]$ with $a = -1$, $b = 1$. Verify that there is no number c for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Explain how this does not violate the mean-value theorem.

6. A certain tollway is 120 miles long and the speed limit is 65 miles per hour. If a driver's entry ticket at one end of the tollway is stamped 12 noon and she exits at the other end at 1:45pm, should she be given a speeding ticket? Explain your answer.