Math 12: Handout DS2
(but write the HW number from the webpage on your assignment)

1. Find the fixed points for all of the following discrete dynamical systems $x_{n+1} = f(x_n)$. Use the Multiplier Theorem to determine the *stability* of the fixed points (are they stable, unstable, or neither?). If the Multiplier Theorem is inconclusive, use graphical analysis to determine your answer.
(a) $f(x) = 1/(x-1)$.
(b) $f(x) = x^2 - 2$.
(c) $f(x) = x^{2/3}$.
(d) $f(x) = \sin(x)$. [Hint: draw a graph].

2. Finding the fixed points of the dynamical system generated by $f(x) = \cos(x)$ is impossible to do analytically, since solving $x = \cos(x)$ in closed form is impossible. However, in this problem you will discover how to locate them anyway.
(a) Can integer multiples of $\frac{\pi}{2}$ be fixed points of this system?
(b) If f had a fixed point p, what would the Multiplier Theorem say as long as p was not a integer multiple of $\frac{\pi}{2}$?
(c) Thus, if p exists, you should be able to find it by iterating f with an appropriate seed. Start with $x_0 = 1$. You can easily iterate this function by pressing the Cosine button on your calculator! What is the numerical value of the fixed point, to 3 decimal places? How many iterations did it take?
(d) Does every initial seed x_0 approach this fixed point? Use graphical analysis to get some intuition, then justify your answer. [Hint: for any x_0, where is $x_1 = f(x_0)$?]

3. Is it possible for a linear dynamical system to have exactly 2 fixed points? Justify.

4. Recall the *logistic map* $f(x) = \lambda x(1-x)$, that models the spread of infection among a population. Let $\lambda = 1$. Find the fixed point of this map. Does the Multiplier Theorem say whether this is a stable fixed point? If not, use graphical analysis to say whether this fixed point is stable, unstable, or stable on one side but not the other.

5. Consider the function $f(\theta) = 3\theta$ where θ is an *angle*. (Thus θ may take on only values in the range $[0, 2\pi)$. For example, $f(3\pi/4) = \pi/4$, since $9\pi/4 = \pi/4$ for angles.)
Find all the fixed points of the dynamical system $\theta_{k+1} = f(\theta_k)$.

Thought Question: [for class next time] The Multiplier Theorem can tell us when a fixed point is stable or not. Is there a way to determine whether a periodic orbit is stable or unstable?

Bonus/Challenge Question: [if you do it, turn in to your professor] Develop a higher derivative test for determining whether a fixed point is stable or not in cases where the Multiplier Theorem is inconclusive. [Hint: use Taylor's Theorem with the derivative form of the remainder.]