

MATH 12: HANDOUT DS4

(BUT WRITE THE HW NUMBER FROM THE WEBPAGE ON YOUR ASSIGNMENT)

1. Let $f(x) = -\frac{13}{12}x^2 + \frac{49}{12}x + 1$.

Verify that $x = 4$ is a periodic point. What is its prime period? What are the other points in its orbit?

Use the multiplier theorem to determine if this a stable or unstable periodic orbit.

2. Define the following dynamical system: let the state space $S = [0, 1]$, all numbers between 0 and 1. Let f be the function which takes the decimal representation, cuts off the first digit, and shifts everything left. [Example: $f(0.1415926\dots) = (.415926\dots)$].

(a) What are all the fixed points of f ? (In other words, which numbers which remain unchanged after applying f once?)

(b) Describe all the periodic points of f of prime period 2?

(c) Describe all points of f which are *eventually periodic* of any period? (Eventually periodic means that the iterates eventually repeat, though they may or may not repeat at first.)

3. Graph the following functions for a range of values of the parameter c to determine whether the bifurcation that occurs is a transcritical, tangent, or period-doubling bifurcation. Using Discrete Tool, estimate the precise value of c in each case where the bifurcation occurs.

(a) $f(x) = ce^x$, as c varies between $c = .3$ and $c = .4$.

(b) $f(x) = x^2 - c$, as c varies between $c = .6$ and $c = .9$. [Focus on the fixed point that is smaller and negative.]

(c) $f(x) = cx(1 - x)^2$, as c varies between $c = .8$ and $c = 1.2$. [Focus on what happens near the fixed point at 0.]

4. Use Discrete Tool to:

(a) estimate the location of the first 4 period-doubling bifurcations for $f(x) = x^2 - \lambda$. For instance, $\lambda_1 = .75$. What is $\lambda_2, \lambda_3, \lambda_4$?

(b) estimate the location of the period 3 “window” in the chaotic region.

5. Water is dripping from a leaky faucet. The time between drips can be modeled by some (unknown) discrete dynamical system. There is a parameter λ associated with a dynamical system which is the tightness (marked by the number of turns) of the faucet handle. At first, you observe that the time between drips is constant at 1 second between drops. As you unscrew the faucet handle past the 1/4-turn mark, the drip times begin to exhibit period 2 behavior (e.g., .7, .9, .7, .9, ... seconds, etc.)

We say that there is a *bifurcation* at $\lambda_1 = 0.25$ turns.

Then as you unscrew the handle past the 1/2-turn mark, it begins to exhibit period 4 behavior. Hence we'll say $\lambda_2 = 0.5$ turns.

Although you don't know the underlying dynamical system, you can still make predictions! Using Feigenbaum's universal constant, *estimate* λ_3 , the position of the handle at which the next period-doubling will occur.

Now try this with a real faucet and see if you can observe period-doubling.