Math 12: Handout DS5
(but write the HW number from the webpage on your assignment)

1. Which of the following exhibit sensitive dependence on initial conditions?
(a) $f(x) = \frac{1}{2}x(1-x)$ on $[0,1]$
(b) $f(x) = x/2$ on $\mathbf{R}$
(c) $f(x) = 5x$ on $\mathbf{R}$
(d) $f(x) = x + 5$ on $\mathbf{R}$

2. Which of the following are *dense* sets on $[0,1]$?
(a) the set $\{0.1, 0.5, 0.8\}$?
(b) the rational numbers in $[0,1]$.
(c) $\{1/2, 1/4, 1/8, 1/16, ...\}$.
(d) *any* fraction in $[0,1]$ with denominator 2^i for some integer i.

3. Recall the *shift map* on decimal numbers between 0 and 1, where f is the function that takes the decimal representation, cuts off the first digit, and shifts everything left. [Example: f(0.1415926...)= (.415926...)].

In this exercise, you will verify that the three conditions of a chaotic system hold in special cases.

(a) If $\delta = .05$, do all points x, y eventually separate by more than δ under iteration by f? Give a brief justification.

(b) Find a periodic point in the interval $I = [0.1125, 0.1126]$.

(c) The orbit of initial seeds in I eventually covers the *entire* interval after how many iterations?

4. Do these exercise: 1, 2, 7, 13 on the last page of your current handout.