Teaching from the Unknown

Synopsis

The goal of teaching is to transform our student's understanding, much as the goal of acting is to transform the audience's reality. In this article we use the context of mathematics to explore connections between teaching and acting and how such connections can help our students learn not only mathematics, but about the nature of mathematics.

One of the central challenges for a good stage performance is to make it all happen for the first time *again and again*. Performers must recreate the text and keep it fresh night after night. An essential technique to accomplish this is to "live in the unknown." Hamlet can't play four and a half hours moping about as if he's going to die. The classroom is also a type of performative space where teachers try to make known ideas come alive, again and again. By "teaching from the unknown" we can also sustain our ability to communicate mathematical ideas with a fresh perspective time after time.

The more I've taught the same material over the years, the more the statements and proofs have become familiar and almost self-evident. What once was mind-expanding (whoa, eigenvalues!) was becoming, dare I say, ordinary (hmm, just eigenvalues). I feared this familiarity was starting to negatively influence my teaching and that some of the freshness and excitement of the ideas was missing. However, if I were to "live in the unknown" and teach from that space I could create an environment where the freshness and excitement would return. Moreover, the longer I teach the more my increased understanding of the material would allow me to better play it from the unknown, just as performers use their depth of understanding of the story to heighten audience engagement, emotional connection, and response.

Many of us naturally teach from the unknown as we reveal our mathematics line by line, step by step. We carefully set the stage and work through the script, building up to the 'aha' moment when the strands of the proof come together for the triumphant conclusion – our denouement. We present from the unknown to engage students in the ideas and let them experience the feeling of creating something *as if for the first time*. Just as performers want the audience to live in the moment and wonder how the play's themes will develop, we too want our students to live in the moment and experience the mathematical ideas unfolding and combining in new ways to create yet more fibers in their mathematical tapestries. By recreating mathematics in this way we let students in on the joy of discovery that permeates the mathematical life. Teaching from the unknown also narrows the gap between teacher and student – given that we both have the same tools on the table, what can we make of the ideas at hand?

This new perspective became particularly clear when I was teaching an honors calculus class for the third year in a row. The course is designed for students who have already completed a year of high school calculus and covers the usual first-semester topics but with a late-transcendental approach. From their prior experience students were familiar with the content but mostly as a jumbled bag of techniques they were anxious to apply. For example, a common proof that $\sin x$ is differentiable leads to the limit of $\sin h/h$ as h tends to 0. "How are we going to compute this limit?" I ask, to which several students offer up an enthusiastic "L'Hopital's rule!" Their enthusiasm wanes as I stand there looking puzzled and perplexed. We had only just introduced the definition of derivative so at this point their "rule" was completely unknown, justifying my bewilderment. "What is this rule you speak of? How does it work?" I ask. Invariably, someone presses through my spurious confusion and suggests we differentiate the numerator, at which point they catch on to the logical inconsistency and students begin to wonder how we can genuinely establish the limit. As another example, given that we followed the late-transcendental approach, I could play the unenlightened professor when students referred to the natural logarithm before we had discussed integration. What is this "natural log" you speak of? Inverse of e^x? What is e? I was living in the unknown which forced students to re-examine their understanding as well.

Students quickly caught on to my game and it changed the way they thought about calculus. By teaching from the unknown they saw that we were not just throwing techniques at problems; we were building a coordinated, consistent, and compelling body of ideas with far-reaching consequences. This approach also shifted the focus from "how" to "why" which added meaningful insight about the nature of mathematics. To advance through their high school curriculum students had concentrated on classifying and solving various calculus problems. While this built their confidence and set up a natural affinity for mathematics, most had not allowed themselves to really question why a given definition is needed, why a given proof works, or why a given theorem is so powerful. Consequently, they were missing a crucial aspect of the mathematical beauty of calculus. Moreover, they were missing out on a meaningful learning experience for what it's actually like to be a mathematician and do mathematics. After all, the unknown is where professional mathematicians live as we seek to uncover new insights and truths.

The calculus course is a slightly unusual example playing off their familiarity with the mechanics of calculus, but most math courses start with some common prior knowledge from which to build new ideas and this creates the potential to teach from the unknown. Sometimes I even purposely misdirect students to downplay a connection in order to heighten the impact when it suddenly reappears to link the formerly disparate ideas. For example, when introducing linear systems of differential equations, uncomplicated examples can motivate the ansatz $\mathbf{y} = e^{\lambda t}\mathbf{v}$ as the form of a solution to the linear system $\mathbf{y}' = A\mathbf{y}$, but students are not expecting to see eigenvalues and eigenvectors appear – that's linear algebra, not differential equations! The subsequent calculation substituting the ansatz into the system that leads to unexpected appearance of $A\mathbf{v} = \lambda\mathbf{v}$ and the magical connection with linear algebra is like a paradigm shifting end of Act I that expands preconceived boundaries and entices the learner further into the adventure.

Can we really compare teacher performance to actor performance? There are certainly many similarities. Like it or not, when class starts, it's showtime. Regardless of classroom pedagogy, teachers still need to create and sustain a process that connects their students with the subject matter. Ann Woodworth, Associate Professor of Theatre at Northwestern University, notes many other similarities:

- Both revolve around the art of communication, including both verbal and nonverbal aspects;

- Both focus on known texts and making the inactive, active;
- Both involve important transitions ("Acts");
- Both are goal oriented: engage the "audience" and develop their interest in the subject matter;
- Both involve co-existing in a shared physical setting with a teacher/performer positioning themselves within or around an "audience."

Before pursuing a career in magic, renowned magician Teller (of Penn & Teller) taught Latin for several years. When asked to reflect on how performance played a role in his teaching he observed "[t]he first job of a teacher is to make the student fall in love with the subject. That doesn't have to be done by waving your arms and prancing about the classroom; there's all sorts of ways to go at it, but no matter what, you are a symbol of the subject in the students' minds" [1]. Central to Teller's educational philosophy is Whitehead's "rhythm of education" which sees intellectual progress proceeding in three stages: romance, precision, and generalization [1, 2]. These stages parallel the progression sought by stage performers who seek to transform audience members from the infancy of their arrival state to the complexity of their exit state with renewed perspective and deeper humanity for the themes explored. Is that not also an aspirational goal for our classes?

Living in the unknown is a crucial technique for stage performers and I believe it can be an equally effective technique for teaching mathematics. Given the common goals of transformation, I wonder what else we can learn from our theater colleagues to positively impact our students' learning.

References

[1] J. Lahey, *Teaching: just like performing magic*, The Atlantic, (2016; Accessed August 15, 2019).

[2] A. N. Whitehead, *The Rhythm of Education*, Christophers, London, 1922.